INTRODUCING ADDITION AND SUBTRACTION SYMBOLS TO FIRST GRADERS

By

SUZANNE MCWHORTER COLVIN

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Suzanne McWhorter Colvin

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It was hypothesized that the time of introduction to addition and subtraction symbols would have no effect on first graders' ability to solve, produce, and interpret number sentences, as well as on their ability to solve story problems. A pretest-posttest comparison group design was used to test this hypothesis. Subjects were 66 first graders at Duval Elementary School, Gainesville, Florida.

Pretests were administered to three separate classrooms measuring subjects' ability to solve, produce, and interpret addition and subtraction number sentences and solve addition and subtraction story problems. There were no statistically significant differences in mean scores among the classrooms ($p > .10$).

Each classroom received a separate treatment. The traditional group was introduced to number sentences before

solving related story problems, the immediate group immediately after solving related story problems, and the delayed group after solving story problems orally for 5 weeks.

Posttests identical to the pretests were administered following the treatment period. Mean scores for the immediate and delayed groups were significantly higher than for the traditional group on the measure of total symbol understanding ($p < .05$). The immediate group had a significantly higher mean score than the delayed and traditional groups on measures of ability to solve and produce number sentences. And the delayed group had a significantly higher mean score on the measure of ability to interpret number sentences.

Four weeks later, subjects were given a second set of posttests identical to the pretests and posttests administered earlier. There were no significant differences on the measure of ability to solve number sentences. The immediate and delayed groups had significantly higher means than the traditional group on measures of ability to produce and interpret number sentences and solve story problems.

Before and after instruction, clinical interviews were conducted to examine strategies employed when solving word problems. Overall, subjects used more problem solving strategies after instruction than before instruction as well as more efficient problem solving strategies.

CHAPTER 1
INTRODUCTION

It seems that even before formal instruction, young children have intuitively formed ideas about addition and subtraction (Hiebert, 1984). Children who are of the ages of 3, 4, and 5 understand that addition increases numerosity and subtraction decreases it (Brush, 1978). They also realize that addition and subtraction are inverse operations (Starkey & Gelman, 1982). Not only do children have an understanding of many of the properties of addition and subtraction prior to formal instruction, they are able to use their understandings to solve addition and subtraction story problems (Carpenter, Hiebert, & Moser, 1981; Ibarra & Lindvall, 1982). In a study by Carpenter, Hiebert, and Moser (1981), first graders were able to solve a variety of story problems employing a variety of strategies before ever receiving instruction in school.

As children proceed through school, they seem to lose their intuitive knowledge of addition and subtraction and their natural ability to solve problems (Wearne & Hiebert, 1984). Previous to formal instruction, first graders tend to use a variety of problem solving strategies to solve problems. After carefully analyzing the problem, they decide upon or invent a strategy which makes sense to them

and use that strategy to solve the problem. After the introduction to formal arithmetic, however, children begin relying upon single strategies learned through formal instruction rather than using their own abilities to analyze and solve problems (Carpenter, Hiebert, & Moser, 1981). Many students blindly apply memorized rules and algorithms which have little or no apparent meaning and hope their answer is correct. Before long, mathematics is viewed as meaningless and incomprehensible with little or no relationship to the outside world (Kamii, 1982; Wearne & Hiebert, 1984). The National Assessment of Educational Progress (1983) reported that by the time students are 17, they perceive mathematics as an application of rules, not necessarily understood, used to get a "right" answer.

It is when children are introduced to the formal symbolism of addition and subtraction that they experience the most difficulty (Ginsburg, 1977; Hiebert, 1981; Hiebert, 1984). While children can be taught to use numerical and operational symbols, there is no assurance that they have an understanding of those symbols. Many children can be proficient in computation without understanding the computational process and the associated symbols (Ginsburg, 1977). For example, Ibarra and Lindvall (1980) found that many primary children able to successfully solve simple addition and subtraction number sentences have difficulty in explaining the meaning of number sentences.

There are many who believe young children are incapable of giving meaning to the arbitrary symbolism of mathematics (Hiebert, 1984; Piaget, 1952) and that introduction to mathematical symbolism should be delayed (Driscoll, 1979; Lovell, 1971). Others have suggested that it is not that children are incapable of understanding the symbolism associated with addition and subtraction, rather that teachers rely upon traditional teaching methods which fail to help form a link between what children already understand and are using and the symbols they are being introduced to in school (Hiebert, 1981).

Typically, children are introduced to addition and subtraction symbolism through instruction on solving number sentences or equations (Campbell, 1984; Hiebert, 1984; Kamii, 1985). Students are presented with a number sentence and are shown a technique, sometimes involving manipulatives or pictures, for solving the number sentence. Instruction on the meaning of individual symbols is usually provided along with the rules for using those symbols (Hiebert, 1984). Quite often, teachers will provide a story problem to go with the number sentence after it has been introduced (Campbell, 1984; Kamii, 1982). In this way, teachers feel that they are helping children to see the relationship between number sentences and the actions represented by those number sentences. Research indicates that this is not the case. Students fail to connect the word problem with the number sentence representing it, treating number

sentences and story problems as if they were completely unrelated (Carpenter, Hiebert, & Moser, 1983; Groen & Resnick, 1977; Hiebert, 1984; Kamii, 1985; Kantowski, 1983; Lindvall & Ibarra, 1981; Wearne & Hiebert, 1984).

Many instructional approaches introduce addition and subtraction symbols before direct experience with objects, actions, or verbal problem solving. Researchers are suggesting that introducing symbols before verbal problems may not be appropriate. They suggest that to be successful in providing a link between the written form and the mathematical concept, teachers must do more than simply relate a verbal problem to a number sentence (Grouws, 1972) or use verbal problems as a means of seeing if students can apply previously learned computational skills (Kantowski, 1981). Instead, they suggest that story problems serve as the context in which addition and subtraction symbols are introduced with equations serving as a representation of the actions associated with the story problems (Campbell, 1984; Hiebert, 1981; Kamii, 1985; Wearne & Hiebert, 1984). Only after a great deal of experience with verbal problems and real objects and events should students be introduced to operational equations used to represent those experiences (Baratta-Lorton, 1976; Hiebert, 1981; Kamii, 1985; Lovell, 1971).

Others agree that the introduction of symbols should occur after direct experiences with objects or verbal problems, but contend that the presentation of the symbols

should immediately follow the experience. Once a student solves a problem verbally or through the manipulation of objects, representation of that problem solving experience should take place by writing a number sentence. That way, frequent links can be made between actions and symbolic representations (Campbell, 1984; Hiebert, 1981; Wearne & Hiebert, 1984).

In order to determine at what point in the instructional process the introduction of symbols should occur, it is necessary to investigate the effects of a variety of instructional approaches within a classroom setting (Weaver, 1982). This study investigated three instructional approaches designed to introduce first grade children to addition and subtraction symbolism. The time at which symbols were introduced during instruction varied with each instructional approach.

Statement of the Problem

Past research indicates that children experience a great deal of difficulty when introduced to the formal symbolism of addition and subtraction. Children seem to lose their intuitive ability to solve problems and are unable to understand and interpret addition and subtraction symbolism (Hiebert, 1981; Hiebert, 1984; Ibarra & Lindvall, 1982; Wearne & Hiebert, 1984). The point at which symbols are introduced during instruction seems to be crucial in

children's understanding of addition and subtraction symbolism. Some researchers contend that the introduction of symbols should be postponed until students have had a variety of experiences with objects, events, and verbal problems (Driscoll, 1979; Lovell, 1971; Piaget, 1952). Others contend that symbols should be introduced immediately following each activity with objects and verbal problems (Campbell, 1984; Hiebert, 1981; Wearne & Hiebert, 1984). And still others endorse introducing addition and subtraction symbols in the traditional manner in which children are first presented with a number sentence, instructed as to how to use the symbols in the number sentence, and offered techniques or procedures for successfully solving the number sentence (Hiebert, 1984). It was the intent of this study to examine the effectiveness of each of these approaches in helping children to successfully solve and interpret addition and subtraction problems and word problems.

More specifically, the purpose of this study was to examine three instructional approaches designed to introduce first graders to addition and subtraction symbolism and the effect of each of these instructional approaches immediately and over time on first graders' ability to do the following:

1. solve written addition and subtraction number sentences;

2. give meaning to addition and subtraction symbols as indicated by the ability to solve, produce, and interpret addition and subtraction number sentences;

3. solve addition and subtraction story problems presented orally.

In addition, the effect of instruction on the processes and strategies children used to solve addition and subtraction story problems was investigated.

Significance of the Study

One of the greatest weaknesses of school mathematics is the failure to provide children with an understanding of the written symbolism of mathematics (Ginsburg, 1977), especially since the majority of school mathematics is comprised of learning to manipulate symbols in computation (Ginsburg, 1977; Greenes, 1981). School arithmetic is based on written numbers and symbols such as addition and subtraction symbols. In order to be successful in arithmetic, children must learn to use symbols properly in computation and must understand symbols (Ginsburg, 1977). Otherwise, mathematics becomes a process of applying meaningless rules and procedures to get the "right answer" with little thinking and reflection on the part of students (Kamii, 1982; Wearne & Hiebert, 1984).

While children can learn to use addition and subtraction symbols very early, they apparently have little or no understanding of those symbols. This is evident in their inability to interpret symbols (Ginsburg, 1977; Hendrickson, 1979; Lindvall & Ibarra, 1980) as well as in their inability to assess the reasonableness of their

answers. Students apply memorized rules and procedures and assume their answer is correct regardless of what the symbols say (DeCorte & Verschaffel, 1981). For example, students are often taught the rule that a plus sign indicates that two numbers should be combined. When students are faced with problems such as 3 + __ = 7, they are very likely to blindly apply the rule they have learned and combine the 3 and the 7 to arrive at an answer of 10. Even as unreasonable as this answer is, many students would not recognize it as such since they had, in their minds, correctly applied the rule.

Before formal instruction, children solve problems by analyzing and interpreting a problem, deciding upon a strategy based upon their interpretation of the problem, and applying the strategy to solve the problem (Carpenter, Hiebert, & Moser, 1981; Hiebert, 1984; Wearne & Hiebert, 1984). If an answer is unreasonable, children will usually notice it, unlike many children who have received formal instruction, because they have carefully analyzed the problem first. As a result, they are able to successfully solve a variety of problems using a variety of strategies and be reasonably sure that their answers are correct (Carpenter, Moser, & Hiebert, 1981).

Because children who have received formal instruction tend to apply memorized rules and procedures without considering the problem structure, they are likely to incorrectly solve problems unfamiliar to them which have no

memorized algorithms that can be directly applied (Kantowski, 1981). The National Assessment of Educational Progress (1981) showed that the majority of students at all age levels could not solve nonroutine problems (problems which have no algorithm that can be applied to guarantee a solution) which required some analysis. This is an indication that while students are learning to use mathematical symbols and prescribed procedures, they have limited understanding of those symbols and procedures.

It is crucial that students be introduced to formal arithmetic in such a way that they are able to use and understand the symbolism while retaining their abilities to analyze and solve problems including unfamiliar or nonroutine problems. Apparently, many of the instructional methods used today are not sufficient in accomplishing this and new instructional techniques must be investigated.

Not only is it crucial to examine how symbols are introduced, it is also important to examine when they are introduced. It has been suggested that verbal problems should serve as a context in which to introduce addition and subtraction symbols (Campbell, 1984; Hiebert, 1984; Wearne & Hiebert, 1984). Questions pertaining to whether it is better to introduce symbols before or after experience with verbal problems should be addressed as well as how soon before or after experiences with verbal problems. Some researchers suggest postponing the introduction of symbols until students have had a great deal of experience with

verbal problems and manipulatives (Driscoll, 1979; Kamii; 1985; Lovell, 1971; Piaget, 1952). Others feel symbolism should be introduced immediately following experiences with verbal problems and concrete manipulatives to establish frequent and clear links between verbal and concrete representations and the written symbolic form (Campbell, 1984; Hiebert, 1984; Lindvall & Cica, 1982; Wearne & Hiebert, 1984). It is necessary to find out at what point in instruction it is better to introduce symbols. This this issue was investigated as a part of this study.

In addition to finding out how methods of instruction affect children's ability to use and understand addition and subtraction symbols, how children's problem solving processes were affected by instruction was examined as well. Educators are increasingly concerned with the way in which children process information and how instruction may affect that processing. Much of the current mathematics research has been more concerned with children's processing of addition and subtraction rather than whether or not children can be taught to correctly solve addition and subtraction problems (Romberg, 1982). Research has focused on how children process information before, during, and after formal instruction (Carpenter, Hiebert, & Moser, 1981). Formal instruction has often been found to discourage processes of analysis, thinking, and interpretation, and encourage processes based upon memory abilities (Hiebert, 1981; Kamii, 1982; Wearne & Hiebert, 1984). It is

important, therefore, to investigate whether certain instructional procedures enhance or interfere with thinking.

Definition of Terms

Addition and Subtraction Symbols

Addition and subtraction symbols refer to written numbers and operational signs of addition and subtraction. The plus sign (+), for example, is the symbol for addition while the minus sign (-) is the symbol for subtraction. Addition and subtraction symbols are usually presented in the form or a number sentence or equation in the form of "a + b = c" or "a - b = c".

Meaning of the Symbol "a + b = c" or "a - b = c"

Meaning of the symbol "a + b = c" or "a - b = c", as defined by Fordham (1974/1975) and Hamrick (1976), refers to the pairing in an individual's mind of the symbol with an action that is appropriate for that symbol. Appropriate actions are those which lead from a representation of the problem to an answer to the question, "How many are represented by the symbol?". An appropriate action for the symbol "5 - 3 = ___" would be to construct a set of five objects and remove a set of three objects to count or identify the remaining objects.

Producing Number Sentences

Producing number sentences involves writing number sentences which accurately represent story problems or

actions with objects. For example, after listening to a story about a child who had 3 marbles and obtained 2 more, a student might produce the number sentence, "3 + 2 = 5". This number sentence would appropriately represent the story problem.

Interpreting Number Sentences

Interpreting number sentences involves telling a story or demonstrating an action which accurately represents a written number sentence. For example, when shown the number sentence, "7 - 4 = 3", a child might tell a story about a friend who had 7 cookies and ate 4 of them and only 3 were left.

Story Problems

Story problems are computational problems presented in story format. In this study, story problems consisted of addition and subtraction story problems. Story problems are also referred to as verbal problems. An example of a simple addition story problem is, "There are three children drawing on the blackboard. If two more children draw on the blackboard, how many children will be drawing on the blackboard altogether?"

Addition Story Problems

Addition story problems are those which require students to use the operation of addition to solve the problem. Carpenter, Hiebert, and Moser (1981) divided addition story problems into the following classes:

1. join problems -- a set increases in quantity as in the following problem: "Danny had 3 pennies. His father gave him 2 more. How many pennies did Danny have altogether?"
2. combine problems -- two or more sets are combined as in the following problem: "Adam has 6 red blocks and 3 blue blocks. How many blocks does he have altogether?"
3. compare problems -- two quantities are compared as in the following problem: "Katie has 3 pieces of candy. Tim has 4 more pieces of candy than Katie. How many pieces of candy does Tim have?"

Subtraction Story Problems

Subtraction story problems are those which require that students use the operation of subtraction to solve the problem. Carpenter, Hiebert, and Moser (1981) divided subtraction story problems into the following classes:

1. separate problems -- a smaller quantity is removed from a larger quantity as in the following problem: "Bill had 7 marbles. He lost 4 of them. How many does he have left?".
2. combine problems -- there is no action direct or implied; two quantities may be considered as parts of a whole as in the following problem: "There are 6 people on the playground. Four are girls. How many are boys?"

3. compare problems -- two quantities are compared as in the following problem: "Martha has 4 kittens. Robin only has 2 kittens. How many more kittens does Martha have than Robin?"
4. equalize problems -- students must decide how to make two quantities equal as in the following problem: "Kathy picked 8 flowers. Lil only had 5 flowers. How many flowers does Lil have to pick to have as many flowers as Kathy has?"

Addition and Subtraction Number Sentences or Equations

Addition and subtraction problems presented in the form of "a + b = c" or "a - b = c" are referred to as number sentences or equations. The letters "a", "b", and "c" represent numerals.

Problem Solving Processes or Strategies

Problem solving processes or strategies are techniques used to solve a problem such as modeling with fingers or objects, using counting sequences, or recalling basic number facts.

Statement of Hypotheses

The following null hypotheses were tested in this study immediately following the treatment period as well as 4 weeks later:

1. The time of introduction of addition and subtraction symbols will have no effect on first

graders' ability to write answers for written addition and subtraction number sentences.

2. The time of introduction of addition and subtraction symbols will have no effect on first graders' ability to produce addition and subtraction number sentences.
3. The time of introduction of addition and subtraction symbols will have no effect on first graders' ability to interpret addition and subtraction number sentences.
4. The time of introduction of addition and subtraction symbols will have no effect on first graders' ability to demonstrate the meaning of addition and subtraction symbols as measured by their ability to solve, produce, and interpret number sentences.
5. The time of introduction of addition an subtraction symbols will have no effect on first graders' ability to solve addition and subtraction story problems presented orally.

Once data were collected, the results were analyzed in order to determine whether the investigation produced evidence that supported the above hypotheses. Any hypothesis tested which was found to be significant at the .05 level of significance or below would be rejected.

CHAPTER 2
REVIEW OF THE LITERATURE

For years, an overriding concern in mathematics education has been the children's learning of arithmetic. Teachers, educational administrators, and parents all expect children to learn to add, subtract, multiply, and divide with efficiency and accuracy. As a result, a great deal of research investigating how children acquire operational concepts and skills has emerged. This is especially true in the area of addition and subtraction, the first two of the four basic operations to be taught in school. The following is a review of the research which examines how children are introduced to and learn addition and subtraction.

The review of the literature is divided into the following sections: (a) Solving Addition and Subtraction Number Sentences, (b) Solving Addition and Subtraction Word Problems, and (c) Initial Instruction on Addition and Subtraction Symbols.

Solving Addition and Subtraction Number Sentences

The majority of the early research on addition and subtraction examined children's ability to compute and was

concerned with problem difficulty (Carpenter & Moser, 1982). There were attempts to rank number facts from most difficult to least difficult so that teachers could better know in what order basic facts should be introduced (Brownwell, 1941). Findings among various studies were inconsistent, however, and as a result no real rankings of the most to the least difficult basic addition and subtraction facts emerged (Carpenter & Moser, 1983). Researchers do agree, however, that in general, subtraction problems are more difficult to solve than addition (Baroody, 1984), problems with larger numbers are more difficult than problems with smaller numbers (Carpenter & Moser, 1983), and problems with a missing addend tend to be among the most difficult to solve (Grouws, 1972; Weaver, 1971).

Current researchers investigating children's understanding of basic number concepts attempt to describe not only what problems can be solved but how those problems are solved as well. Research related to addition and subtraction has focused on what strategies are used to solve problems and how children process addition and subtraction information (Carpenter & Moser, 1982). Methods used to identify these processes usually involve some kind of interviewing technique (Carpenter & Moser, 1982; Steffe, Thompson, & Richards, 1982). Children are individually presented problems, and by observing and recording responses to probing questions, researchers infer how particular problems are solved. There are many limitations to using

interviews, such as the inaccuracy of children's explanations, the effect an interviewer may have on children's problem solving strategies, and the subjective judgment of the experimenter. Regardless of these limitations, the clinical interview is recommended as the most direct and accurate measure of the processes that children use in solving problems (Carpenter & Moser, 1983). From using interviews, several basic addition and subtraction problem solving strategies have been identified.

Addition Strategies

Carpenter and Moser (1982) have identified three levels of addition strategies: strategies based on direct modeling with fingers or physical objects, strategies based on the use of counting sequences, and strategies based on recalled number facts. In the most basic strategy, children use physical objects or fingers to represent each addend and the union of the two addends is counted starting with one. This strategy is called counting all with models and is used frequently by first grade children (Carpenter & Moser, 1982).

Three strategies based on the use of counting sequences have been identified. The first identified by Suppes and Groen (1967) and Groen and Parkman (1972) is called the counting all without models which is similar to the counting all with models except that no physical objects or fingers are used. This requires that children keep track of the number of counting steps in order to know when to stop

counting. Often children will use a rhythmic or cadence counting. The other two strategies are the counting on from first strategy and the counting on from larger strategy. The first of these strategies involves counting forward from the first addend in the problem. The counting sequence begins with the first number given in the problem and continues the number of units represented by the second number. The answer is the final number in the sequence. The counting on from larger strategy is identical except that the child begins counting from the larger addend and continues the number of units represented by the smaller addend. Again, the answer is the final number in the sequence. Sometimes fingers or objects are used with counting strategies to help children keep track of how many units have been counted (Baroody, 1984).

The final addition strategy involves the application of a known or derived addition fact. A derived fact is generated from a small set of known basic facts usually based on doubles or numbers whose sum is 10 (Carpenter & Moser, 1982).

Subtraction Strategies

Several subtraction strategies have also been described by Carpenter and Moser (1983). Among the strategies identified are those involving direct modeling with objects or fingers, those involving counting sequences, and those involving the use of number facts. The following describes each of those strategies.

Direct modeling strategies include the separating from, the adding on strategy, and the matching strategy. The separating from strategy involves a subtractive action where a larger quantity is represented and the smaller quantity is removed from it. With the separating from, children use concrete objects or fingers to construct the larger set, then remove the smaller set counting the objects that remain. For example, 6 - 4 would involve counting out six objects or fingers, counting and removing two of the items, and counting the remaining items to arrive at an answer.

Another strategy using objects or fingers is a strategy involving an additive action called the adding on strategy. With the adding on strategy, a child starts with the smaller quantity and constructs the larger. A number of objects is set out equal to the smaller addend then objects are added to that set until the new collection is equal to the larger addend. Counting the number of objects added gives the solution.

The final strategy involving the use of objects or fingers is the matching strategy. A child puts out two sets of objects and sets are matched one-to-one. Counting the unmatched objects gives the answer. The choice strategy involves a combination of counting down from and counting up from given strategies. In this case, a child decides which strategy requires the fewest number of counts and solves the problem accordingly. This can only be observed when a child is asked to solve several different problems.

Among the strategies involving the use of counting sequences are the counting down from strategy and the counting up from given strategy. With the counting down from strategy, children initiate a backward counting sequence beginning with the larger number and the last number uttered of the counting sequence is the answer. The child must count backwards a certain number of steps. This procedure usually entails a forward count to keep track of the subtrahend or some other device to keep track such as matching the backward count to fingers. In effect, this procedure involves a forward count in order to keep up with how many fingers were counted down. Thus counting down requires two simultaneous processes that in effect go in opposite directions (Baroody, 1984). As a result, the counting down from strategy was found to be very difficult and rarely used by first graders (Carpenter & Moser, 1982).

In the counting up from given strategy, a child initiates a forward counting process beginning with the smaller given number and ends with the larger given number. By keeping track of the number of counting words uttered, the child arrives at an answer. Counting up strategies model a missing addend approach to subtraction (Carpenter & Moser, 1982).

The final subtraction strategy as with addition strategies, is the recalling number facts or using derived facts. These strategies are more often employed by older students as a result of instruction in school. Some first

graders use basic recall of facts if they have had experience with addition and subtraction facts at home or in preschools.

Solving Addition and Subtraction Word Problems

Helping children to become efficient problem solvers is a widely recognized goal of mathematics education today. The National Council of Teachers of Mathematic's (1980) states that problem solving should be the primary focus of school mathematics in the 1980s. Despite the accepted importance of problem solving, many students are not capable of solving relatively straightforward mathematics problems (Carpenter, Mathews, Lindquist, & Silver, 1984; Silver & Thompson, 1984; Zweng, 1979). The problems which seem to present the most difficulty are nonroutine problems (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980) or problems that can not be solved by a routine application of a single arithmetic operation or algorithm (Kantowski, 1981). For example, most children would consider the following a nonroutine problem: "Mary had 9 pieces of candy. She gave her sister 2 pieces and her brother 4 pieces. Her brother gave her 1 piece back. How many pieces of candy does Mary have?" Difficulties in solving these problems seem related to a lack of ability to first analyze and understand a problem before applying a strategy to solve the problem. In addition, memorized algorithms used to solve problems seem to have little or no real meaning.

Most children in elementary school are introduced to problem solving through verbal problems often referred to as word or story problems (Kantowski, 1981). Generally, most problem solving done at the primary level consists of solving one-step word problems (Carpenter, Mathews, Lindquist, & Silver, 1984). A majority of students can solve these problems by mechanically applying a computational algorithm they have learned. Research has shown, however, that even children who have received no instruction can successfully solve problems. This behavior suggests that neither computational skills nor the learning of algorithms are prerequisites for problem solving (Ginsburg, 1977; Groen & Resnick, 1977; Hiebert, 1981; Wearne & Hiebert, 1984). In fact, children may be better problem solvers before instruction than after.

To solve a problem, students should first recognize the structure of the verbal problem, select or invent an appropriate strategy, and correctly apply that strategy (Kantowski, 1981). It seems that prior to instruction, children follow these three steps. Carpenter, Hiebert, and Moser (1983) found that children apply a variety of strategies in solving a variety of problems. In an 8-week study examining the processes that first graders used to solve selected addition and subtraction word problems, they found not only that first graders prior to instruction could solve a variety of story problems using a variety of strategies, but that the strategies chosen were related to

the structure of the problem itself. This indicates that instead of applying a memorized rule or algorithm, children were carefully analyzing the problem first, unlike many children who have received instruction.

Many teachers try to help children in understanding problems by having them look for key words or cues which can suggest what operation to perform. Teachers believe that by providing student with these hints, they are helping them to think about how the variables in a problem are related. Unfortunately, this technique tends to prevent children from thinking about the problem. Instead, they blindly apply the "key-word" approach rather than analyzing the problem first, and choosing or inventing a way to solve the problem (Sowder, 1981).

A number of researchers have examined how the mathematical structure of a task influences problem solving performance. For example, multi-step problems have been found more difficult than one-step problems as were problems requiring the application of more than one operation (Silver & Thompson, 1984). Researchers studying young children's performance on addition and subtraction problems have found that problems indicating an explicit action are easier to solve than problems indicating an implied action (LeBlanc, 1968; Shore & Underhill, 1976). For example, a problem such as "Katie had 4 cookies. Katie's father gave her 3 more cookies. How many cookies does Katie have now?" was found to be easier than a problem such as, "Katie has 4 cookies.

Her father has 3 cookies. How many do they have altogether?". Shores and Underhill (1976) also found that subtraction problems in which the action was "take-away" were much easier to solve than "comparison" or "additive subtraction" problems.

Carpenter and Moser (1983) identified four broad categories of addition and subtraction problems: change, combine, compare, and equalize. Change problems involve actions that cause increases or decreases in some quantity (Riley, Greeno, & Heller, 1983). Two types of change problems are joining problems and separating problems. Joining problems consist of putting together two quantities such as in the problem, "Mary had 4 marbles. Jack gave her 5 more problems. How many marbles does Mary have now?" Separating problems involve decreasing a quantity by the removal of sets such as in the problem, "Mary had 9 marbles and gave 5 marbles to Jack. How many marbles does Mary have left?"

Combine and compare problems involve static relationships for which there is no direct action. Combine problems involve the relationship existing among a set and its subsets. An example of a combine addition problem might be, " Jimmy has 4 blue cars and 2 red cars. How many cars does he have altogether?" An example of a combine subtraction problem might be, "There are 5 people in a car. Two are in the front seat and the rest are in the back seat. How many people are in the back seat?".

Comparison problems involve directly comparing two disjoint sets such as in the subtraction problem, "Bob has 6 cookies and Sue has 8 cookies. How many more cookies does Sue have than Bob?" An example of a comparison addition problem would be, "Bob has 6 cookies. Sue has 2 more cookies than Bob. How many cookies does Sue have?" Equalizing involves changing one of two quantities so that both quantities are equal such as in the problem, "There are 4 girls and 6 boys at a birthday party. How many more girls should come to the party so there are the same number of boys as girls?" Equalize problems have been found to be "awkward" for young children to solve due to the irregular wording of the problems (Carpenter, Hiebert, & Moser, 1983). Often equalize problems are not introduced until children are older.

When examining how first graders solve various word problems before instruction, Carpenter, Hiebert, & Moser (1983) found that children basically use the same pattern for solving addition. This was true after instruction as well. When solving subtraction word problems before instruction, however, first graders were found to use several different strategies for different problem types. Separate problems were primarily solved using a separate strategy, combine problems were solved using separate and add on strategies, and compare problems were usually solved by using a matching strategy as were equalize problems. After instruction, however, all problem types were primarily

solved by applying a single strategy -- the separate strategy.

Somehow, as a result of the instructional process, children move from inventing a variety of modeling and counting strategies to solve a variety of problems, to using one memorized strategy or algorithm. Before instruction children are able to analyze and represent the structure of different problems in order to solve them, and are able to invent sophisticated strategies for solving problems with no aid or dependence upon learned algorithms. Gradually, children become less dependent upon their own abilities to analyze and solve problems, and more dependent upon memorized and mechanical procedures for adding, subtracting, multiplying, and dividing with little regard for the content of the problem (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). It is crucial, therefore, that the influence of instruction be carefully examined. At this point, the effects of instruction are still unclear (Carpenter & Moser, 1983).

Initial Instruction on Addition and Subtraction Symbols

Prior to formal instruction, young children have already developed sound, intuitive ideas about arithmetic operations (Hiebert, 1983). In fact, young children are "surprisingly competent" at simple arithmetic operations and come to first grade with a substantial fund of knowledge.

Even children as young as 4 years old can understand that addition increases numerosity and subtraction decreases numerosity (Brush, 1978) as well as recognize that addition and subtraction are inverse operations in that the effect of one cancels the effect of the other (Gelman & Gallistel, 1978).

As children enter school, they receive instruction in formal arithmetic. It is at this point that many children begin to experience difficulty and replace their sound analytical problem-solving strategies with shallow, meaningless procedures (Hiebert, 1984). Many attribute the difficulty to the abstract formal nature of school mathematics which is so different from the informal mathematics children acquire naturally (Donaldson, 1978; Ginsburg, 1977; Hiebert, 1984). Much of school mathematics involves manipulating symbols according to prescribed rules (Hiebert, 1984) to arrive at a correct answer. Teachers are so concerned with the writing of correct answers, that there is little time left for the development of extension of thinking strategies (Kamii, 1982). As a result, many children do not connect what they learn in school with the mathematical knowledge they already possess (Wearne and Hiebert, 1984). Mathematics becomes a process of applying meaningless rules and procedures with little understanding of what they represent (Kamii, 1982).

Understanding Symbols

Many children experience a great deal of difficulty dealing with symbolic expressions and establishing meaning

for the symbols (Hiebert, 1981). Formal mathematics using abstract notation uses structures very different from those utilized in the informal, natural mathematics of a child (Ginsburg, 1983). As a result, children often do not establish a link between the formal (school mathematics) and the informal (mathematical knowledge acquired naturally) (Ginsburg, 1983; Wearne & Hiebert, 1984). The absence of this link causes a shift from intuitive and meaningful problem solving to mechanical, meaningless problem solving (Hiebert, 1984).

Hiebert (1984) believes the key to success with mathematical symbols is establishing a connection between form (the symbolic phrases) and understanding (intuition and ideas about how mathematics works). He has identified three points in the problem-solving process where form and understanding might be linked. Initially, the symbolic representations in the problem can be linked with referents that give them meaning by association with story problems, real experiences, or concrete objects. Secondly, form and understanding are linked when children connect a procedure or algorithm with the underlying concept. For example, regrouping can be tied to understandings of the base-10 numeration system and place value. And finally, the solution to a problem represented symbolically could be assessed as to its reasonableness or whether it is consistent with other knowledge children have about mathematics.

Concrete materials can be used to represent arithmetic concepts and symbols physically. It is important, however, that links are made between the physical representation and the symbolic representation (Hiebert, 1981). When those links between the physical and the concrete should be made is debated by researchers. Some believe connections should be made immediately while others recommend that children use and manipulate objects for an extended period of time before trying to connect symbols with their actions. (Kamii, 1982; Lovell, 1971; Piaget, 1952).

Simply supplying children with concrete objects is not enough. Children need to use objects and have the opportunity to reflect upon their actions with them in order for an activity to be meaningful (Evans, 1983). It is from their actions with objects and their reflection upon those actions that mathematical knowledge is developed (Piaget, 1952). It is important as well that teachers avoid telling students how to use manipulatives to solve problems; otherwise, manipulatives become no more than "physical algorithms" with little meaning as symbolic algorithms.

Often children are introduced to addition and subtraction through learning how to solve number sentences or equations. They are instructed as to the meaning of individual symbols and are shown a written sentence comprised of those individual symbols (Hiebert, 1984). Some recommend that when first introduced to number sentences, children should use an elementary form of mapping rather

than the traditional form. For example, an arrow can be used instead of the traditional "=". It was found that children understood the implications of an arrow more so than the "=" (Nuffield and Schools Council Project, 1969). Several researchers have found that children have a great deal of difficulty in interpreting "=" and believe it to indicate an action rather than the equality between two quantities (Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980).

Regardless of the form used in writing equations, children have difficulty recognizing the meaning of those equations unless they are tied to real world events and experiences. Many teachers relate number sentences to story problems in an attempt to make this tie (Ibarra & Lindvall, 1982). Grouws (1972) found that to be successful, teachers need to do more than simply provide a story problem to go with an equation after it is introduced. Hiebert (1981) suggested that a variety of experiences need to be provided in writing number sentences that represent verbal problems and writing story problems to represent number sentences.

Problem Solving as a Means of Introducing Addition and Subtraction

Many teachers wait to introduce verbal problems until students can compute and have knowledge of several algorithms. The thinking is that verbal problems provide a means for seeing if students can correctly apply algorithms and computational skills (Kantowski, 1981). It appears that

quite the opposite is true. Students do not necessarily need to know how to compute in order to solve verbal problems. Many students are quite competent at inventing their own strategies for solving problems before learning to "compute" (Carpenter, Moser, & Hiebert, 1979; Ginsburg, 1977; Groen & Resnick, 1977). It seems, in fact, that children need to first be able to comprehend and solve story problems if they are to master simple number sentences (Ibarra & Lindvall, 1982). Therefore, verbal problems may be a more appropriate context in which to introduce operations and thus provide for a variety of interpretations of addition and subtraction as well as a better understanding of the basic operations (Hiebert, 1981).

Campbell (1984) proposed a means of using a problem solving approach to introduce children to addition and subtraction concepts. She suggested that teachers first read a story problem to the students, have students paraphrase the story taking from the story the essentials of the problem, solve for the answer, and finally symbolize the general form that had been described by writing a number sentence and solving it. In this way, symbols are introduced as recording devices that help children in attributing meaning to the symbols. According to Campbell, not only can children acquire computational skills and concepts, they can improve their problem solving abilities as well. While Campbell has proposed introducing symbols immediately after interpreting and solving a story problem,

others have recommended that children have extended experiences with verbal problems first, possibly for several weeks, before introducing the symbols which represent them.

Hamrick (1976) investigated the effects of delaying the introduction of symbols to students. Thirty-eight first graders were classified as "ready" or "not ready" based upon their ability to solve simple addition and subtraction problems verbally. Half of those students classified as "ready" were introduced to symbols immediately and half were introduced to symbols 5 weeks later. The same was done for those students classified as "not-ready". Half were introduced to symbols immediately and half 5 weeks later. Using a measure to assess their understanding of symbols and their ability to solve simple addition and subtraction problems, Hamrick found that students in the "ready" group performed significantly better on the measure of symbol interpretation and understanding than did those in the "not-ready" group who were introduced to symbols immediately. There were no significant differences between those in the immediate group and delayed group among students classified as "ready". There were also no significant differences among groups in their ability to solve addition and subtraction number sentences.

Summary

As is evident in the literature, the way in which children are introduced to addition and subtraction

influences their ability to understand the concepts and processes associated with addition and subtraction, as well as the symbols representing those operations. It also influences the way in which they approach and solve problems. It seems that as a result of the instructional process, children move from inventing and using a variety of problem solving strategies to using a single memorized algorithm. What aspects of the instructional process create this effect, however, is still unclear (Carpenter & Moser, 1983). It is important, therefore, to carefully examine the instructional process and determine what approaches are most appropriate when introducing children to formal arithmetic and how those approaches affect their ability to solve both verbal and written problems and understand the symbols associated with those problems.

When children are introduced to addition and subtraction symbols (number sentences), they seem to experience a great deal of difficulty. Research has indicated that children have a better understanding of symbols if they are introduced after experiences in solving verbal problems with the aid of concrete manipulatives. How soon after these experiences symbols should be introduced is still unclear, however. Should symbols be introduced after several weeks of concrete problem solving experiences, or should they be introduced immediately following concrete experiences as a means of representing or recording those experiences? In addition, it is not clear how the

introduction of symbols affects childrens' ability to solve varying kinds of word or story problems or how the processes used to solve such problems are affected.

In this study, such questions were investigated by examining the effects of three instructional approaches designed to introduce addition and subtraction symbols to first graders. Although all three instructional approaches used word problems as part of the instructional process, the timing of the introduction of symbols is varied. One approach introduced symbols first followed by an associated word problem. The second approach introduced word problems then introduced symbols immediately after as a means of representing the problem. And the third approach introduced symbols after several weeks of solving problems verbally. Chapter III describes the procedures that were followed when conducting the investigation.

CHAPTER 3
RESEARCH METHODOLOGY

Subjects

The subjects for this study were 66 first grade students from Duval Elementary School in Gainesville, Florida. The school is predominantly Anglo-American (approximately 90%) with the majority of students coming from low to middle socioeconomic backgrounds. All three of the first grade classes participated in the study. At the beginning of the school year, students were assigned to classrooms on the basis of sex, race, and reading achievement as measured by the Ginn Basal Reading series to assure an even distribution of boys, girls, Blacks, whites, and students in low, average, and high achieving reading groups in each classroom. Approximately 26 students were assigned to each first grade class. Each classroom was randomly assigned to one of three treatment groups: (a) traditional group, (b) immediate group, and (c) delayed group.

Prior to the treatment period, students were given a variety of numeration tasks to assess their understanding of basic number concepts. Tasks included (a) numeral recognition through nine, (b) rational counting through

nine, (c) recognition of sets through nine, (d) corresponding numerals to sets through nine, (d) number constancy, and (e) understanding of "more" and "less". Only students who mastered all of the above numeration tasks were assigned to treatment groups. All other students received individualized instruction and tutoring by their regular classroom teachers.

Additionally, students who mastered all pretest measures were excluded from treatment groups. These students were assumed to have mastered all of the concepts and skills being presented through each treatment and did not need to receive any of the treatments designed for this study. They too were provided with individualized tutoring by their classroom teachers.

Data Collection Instruments and Procedures

The following is a description of each of the data collection instruments and procedures used in this study. Each instrument and procedure was used in preassessments as well as postassessments. The researcher administered all instruments and conducted all data collection procedures.

The Addition and Subtraction Test

Task One of the measure of symbol understanding was a measure of ability to provide answers for written addition and subtraction number sentences. This task was analyzed separately and used to determine if any differences existed

among treatment groups in ability to solve number sentences (Hypothesis 1).

Measure of Symbol Understanding

As noted in Chapter I, subjects demonstrated that they knew the meaning of a symbol in one of the following two ways: (a) given a symbol, they interpreted that symbol by describing or carrying out an action that is appropriate for the symbol; (b) given an action or description of an action that is appropriate for a symbol, they produced the symbol. When the symbol was an addition or subtraction number sentence, it was also necessary that students stated the correct sum or difference. It was assumed that subjects able to do all of the three tasks described, demonstrated that symbols were more meaningful to them than students who could do only one or two of the above tasks (Hamrick, 1976/1977).

The measure used to assess a subject's understanding of the meaning of symbols (Hypothesis 4) was directly derived from the instrument used by Hamrick (1976/1977) in a study sponsored by the National Science Foundation assessing first graders' understanding of addition and subtraction symbols. The instrument was reviewed by a peer review panel of mathematicians, university professors, and representatives of the National Science Foundation from various locations throughout the U.S. The panel agreed that the instrument was a valid measure of symbol understanding based upon the content of the instrument and the construct being measured.

Test-retest reliability coefficients were computed by the researcher prior to the implementations of the study using a sample of 48 first graders in Gainesville. Each task was administered on one day, then once again on the following day. Pearson product-moment correlation coefficients were calculated for each of the three tasks as well as for the entire instrument showing the relationship between scores on the first administration of the test to scores on the second administration of the test. The reliabilities of task one (solving number sentences), task two (producing number sentences), and task three (interpreting number sentences) were calculated to be .97, .92, and .92, respectively. The reliability of the entire instrument was calculated to be .97.

The instrument was comprised of three tasks: (a) solving written addition and subtraction number sentences, (b) producing written addition and subtraction number sentences, and (c) interpreting addition and subtraction number sentences. Number sentences used in each task were randomly selected from a pool of addition number sentences and a pool of subtraction number sentences whose sums were less than 10 and differences were greater than 1 with no addend greater than 9 or less than 1.

Task one: solving addition and subtraction number sentences. The first task contained 10 written addition number sentences and 10 written subtraction number sentences. Subjects were asked to provide a written answer

for each number sentence. This portion of the test served as the measure of ability to solve addition and subtraction number sentences (Hypothesis 1) as well as one part of the measure of symbol meaning understanding (Hypothesis 4). The assumption was that part of understanding of the meaning of a symbol was knowing how to correctly solve number sentences which use that symbol.

<u>Task two: producing addition and subtraction number sentences</u>. In this task, students observed an action while listening to a story or description and decided whether the action or description represented the operation of addition or subtraction. They then had to decide which symbols represented that operation and write them in the form of a number sentence. For example while manipulating blocks, the examiner might have said, "I have 2 blocks and you have 3 blocks. If you take my 2 blocks and put them with your 3 blocks, you will have 5 blocks." The subject was then asked to write a number sentence appropriate for the action such as, 2 + 3 = 5. Subjects were asked to produce 10 subtraction number sentences and 10 addition number sentences. The results from this task were used to test Hypothesis 2.

<u>Task three: interpreting addition and subtraction number sentences</u>. Subjects were shown 10 addition number sentences and 10 subtraction number sentences each presented separately and in random order. They were then asked to manipulate objects, tell a story, or draw a picture which

represented each number sentence. For example, when presented with the number sentence, 3 + 4 = 7, a child might have told a story like, "John had 3 cookies. His mother gave him 4 more cookies. Then he had 7 cookies." The results from this task were used to test Hypothesis 3.

In order to determine which treatment groups seemed to have had a better understanding of symbols overall, scores of each of the three tasks described were combined and statistically analyzed. These scores were used to test Hypothesis 4.

Assessment of First Graders' Ability to Solve Addition and Subtraction Story Problems.

Two types of addition story problems and three types of subtraction story problems were included in the assessment of ability to solve addition and subtraction story problems presented orally (Hypothesis 5). Addition problem types included combine and compare problems. Join problems were not included because research has shown that children use the same processes to solve join and combine problems (Carpenter, Hiebert, & Moser, 1983). Subtraction problems included separate, combine, and compare problems.

The problems conformed to the following rules: (a) all addends were greater than 1 and less than 10, (b) all sums were less than 10, and (c) the difference of the addends was greater than 1. Four separate problems, 4 combine problems, and 4 compare problems were read to students in random order. After each problem was read, students were asked to

record their answers on a numbered answer sheet. Students were allowed to use manipulatives or to draw pictures to aid them in solving the problems. Test-retest reliability coefficients were obtained by the researcher prior to the implementation of the study using a sample of 48 first graders in Gainesville. The reliability was found to be .99.

Identification of Children's Processes Used to Solve Story Problems.

Identification of the processes and strategies used by students in solving story problems was accomplished through revised clinical interviews. The revised clinical interview was first developed by Piaget when he concluded that the verbal interview method was sometimes inadequate, especially with younger children who often have difficulty expressing themselves verbally. In the revised clinical method, concrete materials can be used. Data of interest in the revised method are both verbalizations and aspects of nonverbal behavior (Ginsburg, Kossan, Schwartz, & Swanson, 1983).

The aim of the interviews was to get an idea of the kind of cognitive processes children employed to solve problems. In other words, what problem solving strategies did children employ to solve verbal story problems and did these strategies differ with the type of problem? While there was a set of procedures to be followed for conducting the revised clinical interview, it was important that there

also be flexibility. In particular, the examiner needed to be able to invent critical tests or questions as the subject responded and to continue to be a neutral influence upon the subject (Ginsburg, Kossan, Schwartz, & Swanson, 1983). Interviews were conducted by the investigator and were audio-taped for future analysis.

Interviews were administered individually to each subject by the researcher. The procedures that were followed when conducting the revised clinical interviews were adapted from Ginsburg, et al. (1983) and were as follows:

1. Initial Presentation of the Task - the interviewer told a story problem randomly chosen from one of the five problem types and asked the subject to show or tell how he/she arrived at a solution to the problem.
2. Concreteness - the interviewer repeated the story emphasizing crucial elements of the story to ensure that the subject had heard and understood the story.
3. The Demand for Reflection - the subject was asked to verbalize or demonstrate his/her thoughts with objects or by acting out or drawing. Questions were asked such as, "How did you get the answer?", "Can you show me how you got that answer?", or "Can you tell me how you got that answer?"

4. Contingency - sometimes the interviewer's questions were contingent upon the subject's response.
5. Standardized Questions - a set of standardized questions was developed to use with each problem type. The order or wording of the questions changed depending upon the response or actions of the subjects.
6. Use of Naturalistic Observation - as the child reacted to questions, observations were made of his/her actions, facial expressions, etc. as possible indicators of his/her thought processes. For example, some children bounced their heads as they solved problems counting their bounces to arrive at an answer.

Procedures

The explanation of the procedures to be followed in implementing this study is divided into three sections: (a) preassessment, (b) treatment conditions, and (c) post-assessment.

Preassessment

During the first week of the study, the following pretests were administered to all subjects:

1. numeration tasks,
2. measure of ability to solve addition and

subtraction number sentences (task one of the measure of symbol understanding),

3. measure of symbol understanding,
4. measure of ability to solve addition and subtraction story problems, and
5. revised clinical interviews identifying first graders' problem solving processes.

Results of the numeration tasks determined what subjects were "ready" to participate in the study. Only students mastering all tasks were deemed as ready to be introduced to addition and subtraction and were included in the study. Students not included in the study were provided with instruction in numeration by their classroom teachers. Results of all other pretests were used to decide which students were too advanced to participate in the study. Students who already knew how to add and subtract, solve word problems, and identify the symbols associated with addition and subtraction did not need to be reintroduced to those concepts and thus did not need to participate in this study. Students who mastered all pretests were considered too advanced and were excluded from participation in treatment groups. They too were provided with instruction by their regular classroom teachers.

Pretest scores also served to determine if any statistically significant differences existed among the three treatment groups in their ability to solve addition and subtraction number sentences and story problems and in

their ability to understand addition and subtraction symbols. Audio-tapes of revised clinical interviews were examined to determine the problem-solving strategies used by first graders before instruction.

Treatment Conditions

Prior to the treatment period, students had received instruction in numeration skills and concepts by their regular classroom teachers. The first grade curriculum for mathematics at Duval Elementary consisted primarily of skills and concepts taught in the Heath Mathematics Series. The following is a list of those skills and concepts and the order in which they were normally presented:

1. numbers 0 through 10
2. addition with sums up to 10
3. subtracting from 10 or less
4. place value
5. time and money
6. sums through 12
7. subtracting from 12 or less
8. geometry and measurement
9. addition and subtraction through two place digits
10. fractions
11. sums through 18 and subtracting from 18

At a meeting with the first grade teachers in August, 1986, it was agreed that no addition and subtraction would be taught by them prior to and during the treatment period. Instead, teachers provided instruction in numeration,

telling time, coin identification, and geometry. The treatment period began once the majority of students had mastered the skills and concepts of numeration.

Each first grade class was randomly assigned to one of three treatment groups. Whole classrooms comprised each treatment group with the exception of those students who did not qualify to be in the study. The following is a description of each treatment group.

The traditional group. In the traditional group, students were instructed on how to solve addition and subtraction number sentences first and were then presented with a story problem to go with each number sentence. Dramatizations and concrete objects were often used when presenting stories. Students first solved number sentences orally and were later asked to solve number sentences by writing answers and number sentences. Problems included all of the five problem types described earlier although the majority of time was spent teaching combine addition and separate subtraction problems. This basic procedure was repeated each day for 5 weeks.

The immediate group. In the immediate group, students were presented with a story problem with the aid of dramatization and concrete objects and were asked to solve the problem orally. Immediately follcwing, a number sentence was introduced which represented the story problem just solved. Immediately following experiences with objects, dramatization, and word problems, students solved

number sentenced orally and were later asked to solve number sentences by writing answers and number sentences. Problems included all of the five problem types described earlier although the majority of time was spent solving combine addition and separate subtraction problems. This basic procedure was repeated for 5 weeks.

The delayed group. Students in the delayed group were presented with story problems with the aid of dramatization and concrete objects and were asked to solve the problems orally. Problems included all of the five problem types described earlier although the majority of time was spent on combine addition and separate subtraction problems. This basic procedure was repeated for 4 weeks. On the fifth week, students were introduced to number sentences as a means of representing their experiences with verbal story problem solving. They continued writing and solving number sentences for the remainder of the week.

Postassessment

The following posttests were administered during the seventh week of the study:

1. measure of ability to solve addition and subtraction number sentences (task one of the measure of symbol understanding - Hypotheses 1 and 4),
2. measure of ability to produce addition and subtraction number sentences (task two of the measure of symbol understanding -- Hypotheses 2 and 4),

3. measure of ability to interpret addition and subtraction number sentences (task three of the measure of symbol understanding -- Hypotheses 3 and 4),
4. measure of overall symbol understanding (combined scores of each of the three tasks on the measure of symbol understanding),
5. assessment of ability to solve addition and subtraction story problems (Hypothesis 5), and
6. revised clinical interviews identifying first graders' problem solving processes.

Tests were scored and analyzed by the investigator to determine if there were any differences among treatment groups. Clinical interviews were analyzed by the investigator to identify the problem solving strategies employed by subjects both before and after instruction.

Four weeks following the completion of the study, a second set of posttests was administered identical to the first set administered directly after the treatment period. Results were again analyzed by the investigator to determine if there were any differences among treatment groups and to see if those differences were similar to those found when analyzing the first set of posttests.

Data Analysis

The research design of this study was a nonequivalent Pretest-Posttest Control Group Design (McMillan &

Schumacher, 1984) using repeated measures (Glass & Hopkins, 1984). Although complete randomization of subjects is ideal, it was neither possible nor practical for this study. As a result, it was necessary to use groups as they had already been organized into classes by using stratified randomization procedures. The school principal had divided students into groups on the basis of sex, race, and reading achievement and had randomly selected students from each group to be assigned to classrooms. Although this procedure had been followed for most students, there were some exceptions such as with students who were repeating a grade. Those students were assigned to classrooms they were not in the previous year.

Pretests were analyzed using an analysis of variance procedure (Glass & Hopkins, 1984) to determine if any significant differences existed among groups before the treatment period. An analysis of covariance procedure was used to analyze both sets of posttests with the pretest serving as the covariate. This procedure was chosen in order to partially compensate for any differences which might have existed in treatment groups before the treatment period (McMillan & Schumacher, 1984). The Bonferroni technique of multiple comparisons was performed as a follow-up procedure to detect where specific mean differences existed.

Audio-tapes of interviews were analyzed to determine which problem solving strategies were used by subjects both

before and after instruction. Groups were then compared to see if subjects in one treatment preferred one strategy more than another as well as to see if problem solving strategies changed more in one treatment group than another.

CHAPTER 4
ANALYSIS AND INTERPRETATION OF THE DATA

The purpose of this study was to examine the effect of three instructional approaches designed to introduce addition and subtraction symbols to first graders. In one instructional approach, first graders were introduced to addition and subtraction symbols in the traditional manner by first teaching them to solve number sentences and then presenting them with related story problems. In the second instructional approach, pupils were first introduced to story problems. Immediately after each story problem was solved a number sentence was introduced to symbolically represent the story problem. In the third and final instructional approach, pupils were presented with story problems to solve orally for 4 weeks. In the fifth week, number sentences were introduced as representations of the story problems solved orally.

First grade classrooms were used at Duval Elementary School in Gainesville, Florida. Each class contained 22 pupils who participated in the study. The study began in September, 1986 and lasted for 7 weeks. During the first week of the study, students were preassessed as to their ability to solve addition and subtraction number sentences, solve addition and subtraction story problems, and produce

and interpret addition and subtraction number sentences. A measure of their total understanding of addition and subtraction symbols was derived by examining their combined ability to solve, produce, and interpret addition and subtraction number sentences. In addition, clinical interviews were conducted to determine what processes or strategies first graders used to solve story problems. The same assessments were administered during the last week of the study following the treatment period. Four weeks later, students were reassessed as to their ability to solve, produce, and interpret number sentences as well as solve story problems.

The purpose of this chapter is to present the data collected as a result of the assessments both before and after the treatment period. These data were statistically analyzed and used to test the five null hypotheses posed in Chapter I. There is also a discussion of the results of the clinical interviews conducted to determine children's processes used to solve addition and subtraction story problems before and after instruction as well as a presentation of the results of the second set of posttests administered 4 weeks following the completion of the study.

Preassessment Results

During the first week of the study, all subjects included in the study were given the following pretests:

1. measure of ability to solve addition and subtraction number sentences,
2. measure of ability to produce addition and subtraction number sentences,
3. measure of ability to interpret addition and subtraction number sentences,
4. total measure of symbol understanding, and
5. measure of ability to solve addition and subtraction story problems.

Mean scores are summarized in Table 1. In all three treatment groups, mean scores were higher for the measure of ability to solve story problems than for the measure of ability to solve number sentences. Across all treatment groups, students were more able to produce number sentences correctly than to interpret number sentences.

Using the scores from the pretests, an analysis of variance was performed to determine if there were any significant differences in treatment groups before the treatment period. Because it was crucial not to make a Type II error and find no significant differences among the treatment groups when indeed there were significant differences, the level of significance was set at .10 rather than .05. F tables can be found in the appendices for both the pretests and posttests.

When examining the ANOVA tables for each pretest, no statistically significant differences were found in mean pretest scores among treatment groups. See Table 2.

Table 1. Mean Scores on Pretests for Each Treatment Group

	Treatment Group		
Pretest	Traditional	Immediate	Delayed
Solving + and - Number Sentences	2.90	3.27	3.50
Producing + and - Number Sentences	3.27	3.77	3.36
Interpreting + and - Number Sentences	1.31	2.14	1.45
Total Symbol Understanding	7.50	9.18	8.51
Solving + and - Story Prolems	5.27	6.13	5.95

As is indicated in Table 2, none of the calculated F values were found to be significant at the .10 level of significance when comparing scores among treatment groups for each pretest. All probability levels were calculated to be higher than the set criterion of .10. As a result, it was concluded that no significant differences existed among treatment groups before the treatment period on any of the pretests administered.

These findings were particularly important due to the fact that complete randomization of subjects was not possible and intact classrooms were used as treatment groups. The fact that the treatment groups were not found to differ significantly before the treatment period indicates that any differences found among treatment groups on the posttests would probably not be due to initial differences in the groups but to other factors.

Clinical interviews were also conducted during the first week of the study to determine what processes subjects used to solve addition and subtraction word problems before instruction. The results of the interviews are discussed in a separate section later in the chapter.

Postassessment Results

Two sets of posttests were administered to subjects during the course of this study. One set was given immediately following the treatment period. A second set

Table 2. Summary of the Analysis of Variance Test of Significance Using Pretest Scores

Pretest	F value	PR > F
Solving + and - Number Sentencs	.24	.78
Producing + and - Number Sentences	.10	.90
Interpreting + and - Number Sentences	.77	.46
Total Symbol Understanding	.20	.81
Solving + and - Story Problems	.19	.82

was given 4 weeks following the treatment period to see if differences in the treatment groups still existed. Posttests were identical to the pretests administered 6 weeks prior and were administered, scored, and analyzed using an analysis of covariance procedure with pretest scores as the covariate. While no statistically significant differences in pretest scores were noted, there were similarities among pretest and posttest scores indicating there may have been initial differences in the groups which could have contributed to differences in posttest scores. For this reason, posttest scores were analyzed using an analysis of covariance procedure, rather than an analysis of variance procedure, because it would partially control for initial differences in treatment groups. The criterion set for statistical significance was .05 (rather than .10 as was the case when analyzing pretests) because it was crucial not to make a Type I error in which significant differences would be found when in reality there were none.

Results of the First Set of Posttests

The first set of posttests were given during the seventh week of the study immediately after the treatment period. The following is a presentation of the findings of each of the posttests administered at that time.

Solving addition and subtraction number sentences

Immediately following the 5-week treatment period, a posttest was administered to all subjects which measured their ability to solve addition and subtraction number

sentences. Students were given 10 addition and 10 subtraction number sentences and asked to write the answer for each. Table 3 shows the mean scores for addition, subtraction, and combined addition and subtraction.

The data showed that the majority of students in each treatment group solved more addition number sentences correctly than subtraction. This is consistent with past research findings revealing that in general, children find subtraction problems more difficult to solve than addition (Baroody, 1984).

The question which was being investigated was whether or not the time of introduction of addition and subtraction symbols would have an effect on students' ability to solve addition and subtraction number sentences. The null hypothesis (Hypothesis 1) was that the timing of the introduction of addition and subtraction symbols would have no effect on first graders' ability to solve addition and subtraction number sentences. In other words, there would be no statistically significant differences among the mean posttest scores for each treatment group. The following statistical null hypothesis was tested:

$$H_o\!: {}_1 = {}_2 = {}_3$$

with ${}_1$ representing the mean for the traditional group, ${}_2$ representing the mean for the immediate group, and ${}_3$ representing the mean for the delayed group.

Mean posttest scores differed numerically, with the immediate group (12.86) having a higher mean score than both

Table 3. Adjusted Mean Posttest Scores for the Measure of Ability to Solve Addition and Subtraction Number Sentences Given Immediately Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Addition	6.58	7.68	6.19
Subtraction	4.14	5.18	4.08
Combined	10.72c	12.86b	10.27c

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

the traditional group (10.72) and the delayed group (10.27). An analysis of covariance was performed to determine if these differences were statistically significant. The level of significance was set at .05.

The computed F value was 4.92 and the null hypothesis was rejected at the .01 level of significance. The conclusion was, therefore, that at least one pair of mean scores was significantly different. Thus, the time of introduction of symbols did seem to have an effect on first graders' ability to solve addition and subtraction number sentences.

In order to detect specifically where significant differences existed, the Bonferroni technique of multiple comparisons was performed. Findings are indicated in Table 3.

No statistical differences were detected when comparing mean posttest scores of the traditional group and the delayed group. Statistical differences were noted, however, when comparing mean posttest scores for the immediate group with those for the traditional and delayed groups. The mean posttest score for the immediate group was found to be significantly higher than scores for both the traditional and delayed groups.

<u>Producing addition and subtraction number sentences</u>

As part of the measure of symbol understanding, first graders were asked to produce number sentences to go with story problems that the researcher told and demonstrated

with manipulatives. The question under investigation was whether or not the timing of the introduction of addition and subtraction symbols would affect first graders' ability to produce number sentences when presented with story problems. The null hypothesis (Hypothesis 2) was that there would be no effect on first graders' ability to produce addition and subtraction number sentences. Consequently, mean posttest scores for each treatment group would not be significantly different. The statistical null hypothesis was in the following form:

$$H_o: \ _1 = \ _2 = \ _3$$

with $_1$ representing the mean for the traditional group, $_2$ representing the mean for the immediate group, and $_3$ representing the mean for the delayed group. The posttest was administered the week following the treatment period to all subjects. Mean scores for each treatment group can be found in Table 4.

As was the case with the scores on the measure of ability to solve addition and subtraction number sentences, scores were lower for addition number sentences than for subtraction number sentences across all treatment groups. Posttest mean scores differed numerically with the traditional group having an adjusted mean score (4.18) lower than adjusted mean scores for both the immediate (8.00) and delayed (5.40) groups. An analysis of covariance was performed to determine if these differences were statistically significant.

Table 4. Adjusted Mean Posttest Scores for the Measure of Ability to Produce Addition and Subtraction Number Sentences Given Immediately Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Addition	2.02	4.36	3.26
Subtraction	2.16	3.64	2.14
Combined	4.18c	8.00b	5.40c

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

The F value was computed at 7.56 ($p = .001$). Thus, the null hypothesis was rejected indicating that for at least one pair of mean scores, the difference was statistically significant.

In order to detect specifically where significant differences existed, the Bonferroni technique of multiple comparisons was performed. As Table 4 indicates, the mean score of the traditional group and the mean score of the delayed group were not significantly different. Mean scores of the traditional and immediate groups were found to differ significantly, however, as were the mean scores of the immediate and delayed groups. Thus the immediate group scored significantly higher on the posttest measuring first graders' ability to produce addition and subtraction number sentences than did the traditional group and the delayed group. The performance of the traditional and delayed groups was statistically equal.

Interpreting addition and subtraction number sentences

Subjects were asked to interpret addition and subtraction number sentences as part of the measure of symbol understanding. Students were individually presented with addition and subtraction number sentences and given a choice of telling a story to go with the number sentences or "acting out" the number sentences with manipulatives. Although, on the pretest, many subjects chose to "act out" number sentences with manipulatives, on the posttest, all subjects chose to tell a story to go with the number

sentences. The results of this task were used to test the null hypothesis (Hypothesis 3) that the time of introduction of addition and subtraction symbols would have no effect on first graders' ability to interpret addition and subtraction number sentences. The following statistical null hypothesis was tested:

$$H_o:\ {}_1 = {}_2 = {}_3$$

with ${}_1$ representing the mean of the traditional group, ${}_2$ representing the mean of the immediate group, and ${}_3$ representing the mean of the delayed group.

Adjusted posttest mean scores can be found in Table 5. Once again, scores were lower for subtraction number sentences than for addition number sentences. The mean score for the traditional group (5.86) was lower than for both the immediate (8.31) and delayed (9.04) groups. In order to determine if this difference was significant, an analysis of covariance was performed with the level of significance set at .05.

The F value was computed at 3.37 ($p = .041$) thus rejecting the null hypothesis and indicating that the difference between at least one pair of mean scores was statistically significant. The Bonferroni method of multiple comparisons was performed in order to determine which pair of means differed statistically. The results are in Table 5.

As is shown in Table 5, the only statistically significant difference was between the mean score for the

Table 5. Adjusted Mean Posttest Scores for the Measure of Ability to Interpret Addition and Subtraction Number Sentences Given Immediately Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Addition	3.49	5.15	5.43
Subtraction	2.37	3.16	3.61
Combined	5.86c	8.31bc	9.04b

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

delayed group and the mean score for the traditional group with the mean score for the delayed group (9.04) being significantly higher than the mean score for the traditional group (5.86). Mean scores for the delayed (9.04) and immediate (8.31) groups did not significantly differ nor did mean scores for the traditional (5.86) and immediate (8.31) groups.

Total measure of symbol understanding

As was noted earlier, it is assumed that students who can perform all three tasks on the measure of symbol understanding (solving addition and subtraction number sentences, producing addition and subtraction number sentences, and interpreting addition and subtraction number sentences) have a better understanding of addition and subtraction symbols than do students who can only do one or two of the above. In order to determine which treatment groups seemed to have had a better understanding of symbols overall, scores of each of the three tasks on the measure of symbol understanding were combined and statistically analyzed. These scores were used to test the null hypothesis (Hypothesis 4) that the time of introduction of addition and subtraction symbols would have no effect on first graders' ability to understand those symbols. In other words, mean scores on the measure of symbol understanding would not significantly differ among treatment groups. The statistical null hypothesis tested was in the following form:

$$H_o: \ _1 = \ _2 = \ _3$$

with $_1$ representing the mean for the traditional group, $_2$ representing the mean for the immediate group, and $_3$ representing the mean for the delayed group. Table 6 contains posttest mean scores for the measure of symbol understanding.

Because mean scores were found to differ numerically, an analysis of covariance procedure was used to determine if those differences were statistically significant with the level of significance set at .05. The computed F value was 3.85 ($p = .026$). The null hypothesis was rejected indicating that there was a statistically significant difference between at least one pair of mean scores.

The Bonferroni method of multiple comparisons revealed that the immediate group (29.17) and the delayed group (24.73) had significantly higher mean scores than the traditional group (20.76) (Table 6). In addition, the mean score for the delayed group did not significantly differ from the mean score for the immediate group.

<u>Solving addition and subtraction story problems</u>

Subjects were also assessed as to their ability to solve addition and subtraction story problems. In each treatment group, students were read 20 story problems by the researcher. Students were given this task in small groups (two groups of 7 and one group of 8). After reading each problem, pupils were asked to write their answer on an answer sheet. Five kinds of story problems were included: combine addition problems, compare addition problems,

Table 6. Adjusted Mean Posttest Scores for the Measure of Total Symbol Understanding Given Immediately Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Solving	10.72	12.86	10.27
Producing	4.18	8.00	5.40
Interpreting	5.86	8.31	9.04
Total	20.76c	29.17b	24.73b

[a] Means with the same letter are not significantly different at the criterion set for statistical signficance (alpha = .05).

separate subtraction problems, combine subtraction problems, and compare subtraction problems.

The hypothesis being tested (Hypothesis 5) was that the time of introduction of addition and subtraction symbols would have no effect on first graders' ability to solve addition and subtraction story problems presented orally. The following statistical hypothesis was tested:

$$H_o: {}_1 = {}_2 = {}_3$$

with ${}_1$ representing the mean for the traditional group, ${}_2$ representing the mean for the immediate group, and ${}_3$ representing the mean for the delayed group. Adjusted mean posttest scores are summarized in Table 7.

The mean posttest score for the traditional group (9.09) was much lower than for both the immediate (13.54) and delayed groups (13.68). An analysis of covariance was performed to determine if this difference was statistically significant at a .05 level of significance. The F value, calculated to be 29.47, was significant at the .0001 level of significance. The null hypothesis was rejected and it was concluded that for at least one pair of mean scores, the difference was significant.

The Bonferroni method of multiple comparisons was used to determine which pairs of mean scores were significantly different. Table 7 summarizes the results of this analysis. While the mean scores for the delayed and immediate groups were not found to differ significantly, the mean score for the traditional group was found to be significantly lower

Table 7. Adjusted Mean Posttest Scores for the Measure of Ability to Solve Addition and Subtraction Story Problems Given Immediately Following the Treatment Period

Treatment Groups	Mean Posttest Scores[a]
Traditional	9.09c
Immediate	13.54b
Delayed	13.68b

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05)

than mean scores for both the immediate and delayed groups. It can be concluded, therefore, that the immediate and delayed groups did significantly better in solving story problems than did the traditional group and that the traditional and delayed groups had mean scores that were statistically equivalent.

Table 8 shows which problem types were more frequently solved correctly for each treatment group. Combine addition story problems were more frequently solved correctly than any other problem type in all three treatment groups followed by separate subtraction problems. In all treatment groups, combine subtraction problems were correctly solved more frequently than both compare addition and compare subtraction story problems but less frequently than combine addition and separate subtraction problems. There were fewer compare subtraction problems solved correctly than any other problem type in all three treatment groups.

Summary of results

After analyzing posttest mean scores the following findings can be reported:

1. The immediate group had a significantly higher mean posttest score on the measure of ability to solve addition and subtraction number sentences than both the delayed and traditional groups. Thus the null hypothesis (Hypothesis 1) was rejected indicating that the time of introduction of addition and subtraction symbols had a significant effect on

Table 8. Number of Each Type of Story Problem Solved Correctly Within Treatment Groups

	Treatment Groups		
Problem Type	Traditional	Immediate	Delayed
Combine Addition	71	83	81
Separate Subtraction	64	81	77
Combine Subtraction	45	74	75
Compare Addition	21	61	89
Compare Subtraction	9	30	25
Total Number Correct	210	329	347

first graders' ability to solve addition and subtraction problems.

2. The immediate group had a significantly higher mean score on the measure of ability to produce addition and subtraction number sentences than both the traditional group and delayed group. Thus the null hypothesis was rejected (Hypothesis 2) indicating that the time of introduction of addition and subtraction had a significant effect on first graders' ability to produce addition and subtraction number sentences.
3. The delayed group had a significantly higher mean score on the measure of ability to interpret addition and subtraction number sentences than the traditional group. Thus the null hypothesis was rejected (Hypothesis 3) indicating that the time of introduction of addition and subtraction symbols had a significant effect on first graders' ability to interpret addition and subtraction number sentences.
4. Both the immediate and delayed groups scored significantly higher on the measure of total symbol understanding than the traditional group. Thus the null hypothesis was rejected (Hypothesis 4) indicating that the time of introduction of addition and subtraction symbols had a significant effect on first graders' ability to understand the meaning of addition and subtraction symbols.

5. Both the immediate group and the delayed group scored significantly higher on the measure of ability to solve addition and subtraction story problems. Thus the null hypothesis was rejected (Hypothesis 5) indicating that the time of introduction of addition and subtraction symbols had a significant effect on first graders' ability to solve addition and subtraction story problems.

Results of the Second Set of Posttests

Four weeks following the completion of the study, all subjects were given a second set of posttests identical to both the pretests and first set of posttests administered immediately following the treatment period to determine if differences found to exist immediately after the treatment period were maintained. The following is a presentation of the results of each assessment. Data again were analyzed using an analysis of covariance with the pretest as a covariate.

Solving addition and subtraction number sentences

Table 9 contains the adjusted mean scores for the measure of first graders' ability to solve addition and subtraction number sentences. Mean scores on this posttest were numerically higher for all treatment groups than mean scores on the posttest administered immediately following the treatment period. Thus, subjects solved more number sentences correctly on the second set of posttests given 4 weeks after the treatment period than on the first set of posttests.

Table 9. Adjusted Mean Posttest Scores for the Measure of Ability Solve Addition and Subtraction Number Sentences Given Four Weeks Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Addition	8.22	9.18	8.13
Subtraction	7.13	7.54	6.73
Combined	15.35b	16.72b	14.86b

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

An analysis of covariance revealed that mean scores on the measure of ability to solve addition and subtraction number sentences did not significantly differ among treatment groups when administered the second time. The calculated F value was 1.50 at a probability level of .23 which is greater than the criterion for statistical significance set at .05. This differed from the results of the posttest given just after the treatment period in which there were statistically significant differences among mean scores with the immediate group performing significantly better than both the delayed and traditional groups.

Producing addition and subtraction number sentences

Results of the measure of first graders' ability to produce addition and subtraction number sentences administered 4 weeks following the completion of the study are summarized in Table 10. Once again, mean scores were higher across all treatment groups for this posttest than for the same posttest administered 4 weeks prior. Mean scores for the second posttest differed among treatment groups with the mean score for the delayed group being higher than mean scores for both the immediate and traditional groups and the mean score for the immediate group being higher than the mean score for the traditional group. An analysis of covariance was performed to determine if these differences were significant.

The computed F value of 7.05 was found to be significant ($p = .0017$). Thus at least one pair of mean

Table 10. Adjusted Mean Posttest Scores for the Measure of Ability to Produce Addition and Subtraction Number Sentences Given Four Weeks Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Addition	5.55	6.95	7.45
Subtraction	4.45	6.09	6.77
Combined	10.00c	13.04b	14.22b

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

scores was found to be significantly different. The Bonferroni technique of multiple comparisons showed that the delayed (14.22) and immediate (13.04) groups performed significantly better on the measure of ability to produce addition and subtraction number sentences than the traditional group (10.00) and the delayed and immediate groups did not significantly differ.

Results from the analysis of the posttest measuring first graders' ability to produce addition and subtraction number sentences given just after the treatment period differed from the results of the posttest given 4 weeks later. As was described in Table 4, the mean score for the immediate group (8.00) was significantly higher than mean scores for both the traditional (4.18) and delayed (5.40) groups just after the treatment period. In addition, the mean score for the delayed group was not significantly different from the mean score for the traditional group. While on the first posttest, the delayed group did not perform as well as the immediate group and equal to the traditional group, the delayed group performed significantly better than the traditional group and statistically equal to the immediate group on the posttest administered 4 weeks later.

Interpreting addition and subtraction number sentences

Mean scores of the measure of ability to interpret addition and subtraction number sentences given 4 weeks after the completion of the treatment period can be found in Table 11.

Table 11. Adjusted Mean Posttest Scores for Measure of Ability to Interpret Addition and Subtraction Number Sentences Given Four Weeks Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Addition	4.77	6.82	6.54
Subtraction	3.82	5.18	5.64
Combined	8.59c	12.00b	12.18b

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

Mean scores for all treatment groups were higher than were mean scores for the posttest administered 4 weeks prior. Mean scores on this posttest differed among treatment groups with the traditional group having a lower mean score (8.59) than both the immediate group (12.00) and the delayed group (12.18). An analysis of covariance was performed to determine if there were any statistically significant differences.

The computed F value of 6.43 was found to be significant ($p = .0029$) indicating that there was a significant difference among at least one pair of mean scores. The Bonferroni technique of multiple comparisons revealed that the mean scores for the delayed and immediate groups were significantly higher than the mean score for the traditional group (Table 11). Mean scores for the delayed group and immediate group did not significantly differ.

These results differed from the results of the posttest administered 4 weeks prior. The only significant difference noted just after the treatment period was between the delayed group and the traditional group with the mean score for the delayed group (9.04) being significantly higher than the mean score for the traditional group (5.86).

Total measure of symbol understanding

The results of the second set of posttests measuring total symbol understanding are summarized in Table 12. Mean scores were again higher for all treatment groups for the posttests given 4 weeks after the treatment period than for

Table 12. Adjusted Mean Posttest Scores for the Measure of Total Symbol Understanding Given Four Weeks Following the Treatment Period

	Treatment Group[a]		
Test	Traditional	Immediate	Delayed
Solving	15.35	16.72	14.86
Producing	10.00	13.04	14.22
Interpreting	8.59	12.00	12.18
Total	33.95c	41.77b	41.27b

[a] Means with the same letter are not significantly different at the criterion set for statistical signfi-cance (alpha = .05).

the posttests given 4 weeks prior. The immediate and delayed groups produced mean scores on the posttest given 4 weeks after the treatment period that were higher than mean scores for the traditional group. An analysis of covariance found this difference to be a statistically significant difference ($F_{2,63} = 4.94$, $p = .01$).

The Bonferroni technique of multiple comparisons found the difference in mean scores for the immediate (41.77) and delayed groups (41.27) not to be statistically significant. Mean scores for the traditional group (33.95) and the immediate group, however, as well as mean scores for the traditional group and the delayed group, were significantly different. Mean scores for the immediate and delayed groups were found to be statistically higher than the mean score for the traditional group (Table 12). These results were consistent with the results of the first set of posttests measuring total symbol understanding administered immediately following the treatment period.

Solving addition and subtraction story problems

The results of the measure of ability to solve story problems given 4 weeks after the treatment period are summarized in Table 13 and were similar to the results of the posttest administered immediately following the treatment period. While mean scores were lower across all treatment groups for the second posttest than for the first, the immediate (8.81) and delayed (9.81) groups continued to produce higher mean scores than the traditional group

Table 13. Adjusted Mean Posttest Scores for the Measure of Ability to Solve Addition and Subtraction Story Problems Given Four Weeks Following the Treatment Period

Treatment Groups	Mean Posttest Scores[a]
Traditional	4.27c
Immediate	8.81b
Delayed	9.81b

[a] Means with the same letter are not significantly different at the criterion set for statistical significance (alpha = .05).

(4.27). The analysis of covariance indicated that a significant difference existed between at least one pair of mean scores ($F_{2,63} = 8.77$, $p = .0004$). Using the Bonferroni technique of multiple comparisons, it was determined that both the immediate and delayed groups had significantly higher scores than the traditional group (Table 13). No statistically significant differences were found to exist between mean scores for the immediate and delayed groups. This was true as well when results from the first posttest given 4 weeks prior were analyzed.

Summary of results

After analyzing mean scores of the second set of posttests administered 4 weeks following the treatment period, the following findings can be reported:

1. On the measure of ability to solve addition and subtraction number sentences, there were no significant differences among treatment groups. On the posttest administered immediately after the treatment period, however, the immediate group had a significantly higher mean score than the delayed group and the traditional group.
2. On the measure of ability to produce number sentences, the mean scores of the delayed and immediate groups were statistically equal and significantly higher than the mean score of the traditional group. On the posttest administered 4 weeks prior, the immediate group had a significantly higher means than both the delayed

group and the traditional group whose means were statistically equivalent.

3. On the measure of ability to interpret number sentences, both the immediate and delayed groups had significantly higher mean scores than the traditional group. This differed from the results of the posttest given just after the treatment period in which the delayed group had a significantly higher mean score than the traditional group.
4. Results of the measure of total symbol understanding were the same for both the posttest given 4 weeks after the treatment period and the posttest given immediately after the treatment period with the immediate group and the delayed group performing significantly better than the traditional group.
5. Results of the measure of ability to solve story problems were the same for both the posttest given 4 weeks after the treatment period and the posttest given immediately after the treatment period. The immediate and delayed groups had significantly higher mean scores than the traditional group.
6. All mean scores for the second set of posttests were higher than scores for the first set of posttests given 4 weeks prior except for the measure of ability to solve addition and

subtraction story problems. Mean scores on this measure were lower for the second posttest than for the first posttest.

Clinical Interviews

During the first week and the last week of the study, subjects were individually interviewed. At the time of the first interview, no formal instruction had occurred. At the time of the second interview, children had received 5 weeks of instruction. The kind of instruction each subject received varied depending upon the treatment group (traditional, immediate, or delayed) to which each subject had been assigned. During the interviews, subjects were asked to solve addition and subtraction story problems and explain or demonstrate how they had arrived at their solutions. There were five problem types included: combine addition, combine subtraction, separate subtraction, compare addition, and compare subtraction. Results of the interviews before instruction are summarized in Table 14 and after instruction in Table 15.

Solving Combine Addition Problems

Combine addition problems, as pointed out earlier, involve combining or joining two or more separate sets such as in the problem, "John had 4 cookies. His mom gave him 3 more cookies. How many cookies does John have altogether?" Table 16 shows the problem solving strategies used both

Table 14. Problem Solving Strategies Employed Before Instruction

Strategy	Trmt Grp	Problem Combine Add.	Combine Sub.	Separate Add.	Compare Add.	Compare Sub.
Counting All With Models	T	12	2	0	13	0
	I	15	0	0	11	0
	D	15	0	0	10	0
	Total	42	2	0	34	0
Counting All Without Models	T	0	0	0	0	0
	I	0	0	0	0	0
	D	0	0	0	0	0
	Total	0	0	0	0	0
Counting Up From First	T	7	0	0	3	0
	I	7	0	0	4	0
	D	4	0	0	6	0
	Total	18	0	0	13	0
Adding On	T	0	5	2	0	0
	I	0	2	1	0	0
	D	0	4	2	0	0
	Total	0	11	5	0	0
Separate From	T	0	10	17	0	2
	I	0	16	15	0	4
	D	0	13	15	0	2
	Total	0	39	47	0	8
Matching	T	0	1	0	0	7
	I	0	1	0	0	8
	D	0	0	0	0	8
	Total	0	2	0	0	23
Basic Facts	T	2	0	0	0	0
	I	0	0	0	0	0
	D	0	0	0	0	0
	Total	2	0	0	0	0
Other	T	1	4	3	6	13
	I	0	3	6	7	10
	D	3	5	5	6	12
	Total	4	12	14	19	35

Table 15. Problem Solving Strategies Employed After Instruction

Strategy	Trmt Grp	Problem: Combine Add.	Combine Sub.	Separate Sub.	Compare Add.	Compare Add.
Counting All With Models	T	11	2	0	11	0
	I	3	0	0	5	0
	D	4	0	0	6	0
	Total	18	2	0	22	0
Counting All Without Models	T	1	0	0	2	0
	I	2	0	0	4	0
	D	2	0	0	3	0
	Total	5	0	0	9	0
Counting Up From First	T	6	0	0	3	0
	I	12	0	0	9	0
	D	13	0	0	10	0
	Total	31	0	0	22	0
Adding On	T	0	1	2	0	0
	I	0	3	2	0	2
	D	0	4	6	0	2
	Total	0	8	10	0	4
Separate From	T	0	10	15	0	7
	I	0	14	16	0	4
	D	0	12	14	0	3
	Total	0	36	45	0	14
Matching	T	0	0	0	0	5
	I	0	1	0	0	9
	D	0	1	0	0	6
	Total	0	2	0	0	20
Basic Facts	T	4	2	2	0	0
	I	3	2	4	0	0
	D	5	4	2	0	0
	Total	12	8	8	0	0
Other	T	0	7	2	6	10
	I	0	2	1	4	7
	D	0	1	0	3	11
	Total	0	10	3	3	28

Table 16. Problem Solving Strategies Used When Solving Combine Addition Problems Before and After Instruction

		Treatment Group				
Strategy	Time[a]	Traditional	Immediate	Delayed	Total	# Correct[b]
Counting All With Models	B	12	15	15	42	22
	A	11	3	4	18	14
Counting All Without Models	B	0	0	0	0	--
	A	1	2	2	5	1
Counting Up From First	B	7	7	4	18	10
	A	6	12	13	31	26
Adding On	B	0	0	0	0	--
	A	0	0	0	0	--
Separate From	B	0	0	0	0	--
	A	0	0	0	0	--
Matching	B	0	0	0	0	--
	A	0	0	0	0	--
Basic Facts	B	2	0	0	0	2
	A	4	3	5	12	11
Other	B	1	0	3	4	0
	A	0	0	0	0	--

[a] B = before instruction
A = after instruction

[a] Total Correct Before Instruction = 34
Total Correct After Instruction = 52

before and after instruction in solving combine addition problems.

Out of the 66 subjects interviewed, 42 used the counting all with models strategy to solve the combine addition problem before receiving instruction. Eighteen used the counting up from first strategy, 2 used the basic facts strategy, and 4 used an unidentifiable strategy such as guessing or "thinking". Thus most subjects chose to use the counting all with models strategy across all treatment groups to solve the combine addition problem before receiving instruction. Those that did not use the counting all with models strategy, primarily used the counting up from first strategy. This again was true in all treatment groups. The only other strategy used, basic facts, was used by 2 subjects in the traditional group. They reported that their parents had taught them to add.

Thirty-four subjects solved the combine addition problem correctly. So just over half of the subjects tested solved the combine addition problem correctly before receiving instruction. Of those 34 subjects, 13 used the counting up from first strategy. Only 5 subjects using this strategy did not solve the problem correctly. Both subjects using the basic facts strategy solved the problem correctly and 14 out of the 42 subjects using the counting all with models strategy solved the problem correctly.

The results of the interviews conducted after instruction differed somewhat. All subjects used an

identifiable strategy unlike before when 4 subjects had not. The strategy used by more subjects in the immediate and delayed groups was the counting up from first strategy with 13 subjects in the delayed group and 12 subjects in the immediate group using this strategy. Only 6 subjects in the traditional group used this strategy.

The strategy used by more subjects in the traditional group continued to be the counting all with models strategy with 11 subjects using this strategy. Only 3 subjects in the delayed group used this strategy and 4 in the immediate group.

The basic facts strategy was used by more subjects in the second interview than in the first. Four subjects in the delayed group, 3 subjects in the immediate group, and 5 subjects in the traditional group used the basic facts strategy to solve the combine addition problem during the second interview compared to 2 subjects in the first interview who used the basic facts strategy.

A strategy not previously used was the counting all without models strategy. Two subjects in the delayed group and 2 in the immediate group used this strategy during the second interview. There was 1 subject in the traditional group who used this strategy.

The number of subjects solving the combine addition problem correctly after instruction (52) was higher than before instruction (34). Fourteen out of the 18 subjects using the counting all with models strategy, 1 out of the 5

using the counting all without models strategy, 26 out of the 31 using the counting up from first strategy, and 11 out of the 12 subjects using the basic facts strategy solved the problem correctly after instruction.

To summarize, when solving combine addition problems before instruction, three identifiable strategies were employed: counting all with models, counting up from first, and basic facts. When solving combine addition problems after instruction, four identifiable strategies were employed in the delayed and immediate groups: counting all with models, counting up from first, basic facts, and counting all without models. The traditional group continued to use only three identifiable strategies. Overall, more subjects began using the counting up from first strategy and the basic facts strategy after receiving instruction. This was especially true in the immediate and delayed groups. In addition, more subjects solved the problem correctly after instruction than before instruction with subjects using the basic facts strategy producing the highest number of correct responses.

Solving Combine Subtraction Problems

Combine subtraction problems are problems in which there is no direct or implied action such as in the following problem: "There are 6 kids on the playground. Two are girls. How many are boys?" Table 17 shows the problem solving strategies used before and after instruction when solving the combine subtraction problem.

Table 17. Problem Solving Strategies Used When Solving Combine Subtraction Problems Before and After Instruction

		Treatment Group				
Strategy	Time[a]	Traditional	Immediate	Delayed	Total	# Correct[b]
Counting All With Models	B	2	0	0	2	0
	A	2	0	0	2	0
Counting All Without Models	B	0	0	0	0	--
	A	0	0	0	0	--
Counting Up From First	B	0	0	0	0	--
	A	0	0	0	0	--
Adding On	B	5	2	4	11	7
	A	1	3	4	8	7
Separate From	B	10	16	13	39	11
	A	10	14	12	36	23
Matching	B	1	1	0	2	2
	A	0	1	1	2	1
Basic Facts	B	0	0	0	0	--
	A	2	2	4	8	7
Other	B	4	3	5	12	0
	A	7	2	1	10	0

[a] B = before instruction
A = after instruction

[b] Total Correct Before Instruction = 20
Total Correct After Instruction = 38

When asked to solve a combine subtraction problem both before and after instruction, most subjects used the separating from strategy across all treatment groups. Thirty-nine subjects used the separating from strategy before receiving instruction and 36 after receiving instruction.

The adding on strategy was the second most commonly used strategy when solving the combine subtraction problem. 11 subjects used the strategy before instruction and 8 after instruction. More subjects in the traditional group used this strategy before instruction (4) than after instruction (1).

Two subjects before and after instruction used the matching strategy. In both cases, the subjects made a set with all of the children on the playground, then a set of the two girls below it. Realizing there were four "left-overs" in the original set of all children on the playground, subjects responded that there were 4 boys "left over".

While there were no subjects that used the basic facts strategy before instruction, 8 subjects used this strategy after instruction. Four subjects were in the delayed group, 2 in the immediate group, and 2 in the traditional group.

Twelve subjects used "other" unidentifiable strategies to solve the problem before instruction. Most of these strategies involved guessing. Some subjects when asked to explain how they arrived at their answer simply said that

they solved the problem by "thinking". Ten subjects used "other" strategies after receiving instruction with 7 of those subjects belonging to the traditional group. Only 1 subject in the delayed group and 2 subjects in the immediate group used unidentifiable strategies to solve combine subtraction problems following the instructional period.

There were 2 subjects in the traditional group who used the counting all with models strategy incorrectly to solve this problem. They combined sets to arrive at an incorrect response. The same 2 subjects used this strategy before and after instruction. It appears that both subjects assumed the problem to be an addition problem.

More subjects responded correctly to the problem after instruction (38) than before instruction (20). Before instruction, 7 out of the 11 subjects using the adding on strategy, 11 out of the 39 subjects using the separate from strategy, and 2 out of the 2 subjects using the matching strategy solved the problem correctly. None of the subjects using the counting all with models strategy or an unidentifiable strategy solved the problem correctly.

After instruction, 7 out of 8 subjects using the adding on strategy, 23 out of 36 subjects using the separate from strategy, 1 out of 2 subjects using the matching strategy, and 7 out of 8 subjects using the basic facts strategy solved the problem correctly. Again, all subjects using the counting all with models strategy or an unidentifiable strategy solved the problem incorrectly.

To summarize, when solving combine subtraction problems, more subjects chose to use the separating from strategy more than any other strategy. The second most commonly used procedure during the first interview was the adding on strategy while during the second interview the second most commonly used procedures included both the adding on strategy and the basic facts strategy. In the immediate and delayed groups, fewer subjects used unidentifiable strategies in the second interview than in the first. The traditional group, on the other hand, had more subjects using unidentifiable strategies during the second interview than in the first. In addition, more subjects solved the problem correctly after instruction than before, especially subjects using the adding on or basic facts strategy.

Solving Separate Subtraction Problems

Separate subtraction problems are problems in which a smaller quantity is removed from a larger quantity as in the following problem: "Paul had 7 pieces of candy. He gave 3 pieces to Mary. How many pieces of candy does Paul have left?". Table 18 shows what strategies were used in solving the separate subtraction problem before and after instruction.

Most subjects used the separating from strategy to solve the separate subtraction problem. Forty-seven out of 66 subjects chose the separating from strategy during the first interview and 45 subjects chose the separating from

Table 18. Problem Solving Strategies Used When Solving Separate Subtraction Problems Before and After Instruction

		Treatment Group				
Strategy	Time[a]	Traditional	Immediate	Delayed	Total	# Correct[b]
Counting All With Models	B	0	0	0	0	--
	A	0	0	0	0	--
Counting All Without Models	B	0	0	0	0	--
	A	0	0	0	0	--
Counting Up From First	B	0	0	0	0	--
	A	0	0	0	0	--
Adding On	B	2	1	2	5	5
	A	2	2	6	10	7
Separate From	B	17	15	15	47	20
	A	15	16	14	45	32
Matching	B	0	0	0	0	--
	A	0	0	0	0	--
Basic Facts	B	0	0	0	0	--
	A	2	4	2	8	8
Other	B	3	6	5	14	0
	A	2	1	0	3	0

[a] B = before instruction
A = after instruction

[b] Total Correct Before Instruction = 25
Total Correct After Instruction = 47

strategy during the second interview following the treatment period. The only other strategy used during the first interview was the adding on strategy. Five subjects used this strategy.

During the second interview, 10 subjects used the adding on strategy with 6 of those subjects in the delayed group, 2 in the immediate group, and 2 in the traditional group. The only other strategy used during the second interview was the basic facts strategy. Eight subjects used this strategy after receiving instruction. Four subjects were in the immediate group, 2 in the delayed group, and 2 in the traditional group. Only 3 subjects used an unidentifiable strategy during the second interview versus 14 during the first interview.

More subjects answered the problem correctly after instruction (48) than before instruction (25). Five out of the 5 subjects using the adding on strategy and 20 out of the 47 subjects using the separate from strategy solved the problem correctly before instruction. After instruction, 7 of the 10 subjects using the adding on strategy, 32 out of the 45 subjects using the separate from strategy, and 8 of the 8 subjects using the basic facts strategy solved the problem correctly. As was the case before instruction, no subjects using unidentifiable strategies solved the problem correctly.

To summarize, the majority of subjects both before and after instruction chose to use the separating from strategy

when solving the separate subtraction problem. More subjects used the adding on and basic facts strategies after instruction than before instruction. In addition, more subjects after instruction than before instruction, responded correctly to the problem with subjects using the adding on and basic facts strategy producing more correct answers than subjects using other strategies.

<u>Solving Compare Addition Problems</u>

Compare addition problems involve comparing two quantities as in the problem, "Andy has 5 stickers. Linda has 3 more stickers than Andy. How many stickers does Linda have?". Table 19 shows the problem solving strategies used before and after instruction when solving compare addition problems.

Before receiving instruction, most subjects used the counting all with models strategy to solve the compare addition problem. Ten subjects in the delayed group used this strategy, 11 in the immediate group, and 13 in the traditional group. The second most commonly used strategy during the first interview was the counting up from first strategy. Six subjects in the delayed group, 4 in the immediate, and 3 in the traditional group used this strategy. It is interesting to note that 19 subjects did not use an identifiable strategy when solving the problem indicating that they probably did not know how to solve the problem. Six subjects were in the delayed group, 7 in the immediate group, and 6 in the traditional group.

Table 19. Problem Solving Strategies Used When Solving Compare Addition Problems Before and After Instruction

		Treatment Group				
Strategy	Time[a]	Traditional	Immediate	Delayed	Total	# Correct[b]
Counting All With Models	B A	13 11	11 5	10 6	34 22	10 15
Counting All Without Models	B A	0 2	0 4	0 3	0 9	-- 4
Counting Up From First	B A	3 3	4 9	6 10	13 22	7 12
Adding On	B A	0 0	0 0	0 0	0 0	-- --
Separate From	B A	0 0	0 0	0 0	0 0	-- --
Matching	B A	0 0	0 0	0 0	0 0	-- --
Basic Facts	B A	0 0	0 0	0 0	0 0	-- --
Other	B A	6 6	7 4	6 3	19 13	0 0

[a] B = before instruction
A = after instruction

[b] Total Correct Before Instruction = 17
Total Correct After Instruction = 31

Results from the second interview were somewhat different. More subjects chose to use the counting up from first strategy in the immediate and delayed groups rather than the counting all with models strategy. Ten subjects in the delayed group and 9 subjects in the immediate group chose to use the counting up from first strategy whereas before only 6 subjects in the delayed group and 4 in the immediate group used this strategy. Three subjects used this strategy in the traditional group during the second interview just as they had done in the first interview.

Another difference found between the first interview and the second interview was the number of subjects who used unidentifiable strategies. Fewer subjects in both the delayed and immediate groups used unidentifiable strategies than in the traditional group. Only 3 subjects in the delayed group and 4 subjects in the immediate group used an unidentifiable strategy versus 6 for the traditional group. The number of subjects in the traditional group using an unidentifiable strategy did not change from the first interview to the second.

The remaining subjects used either the counting all with models strategy or counting all without models strategy during the second interview. Six subjects in the delayed group used the counting all with models strategy and 3 subjects used the counting all without models strategy. In the immediate group, 5 subjects used the counting all with models strategy and 4 used the counting all without

models. In the traditional group, the majority of subjects used the counting all with models strategy as they did in the first interview. Only 2 subjects used the counting all without models strategy and 11 used the counting all with models strategy.

Once again, more subjects solved the problem correctly after instruction (31) than before instruction (17). Before instruction, 10 out of the 34 subjects using the counting all with models strategy and 7 out of the 13 using the counting up from first strategy solved the problem correctly. After instruction, 15 out of 22 subjects using the counting all with models strategy, 4 out of 9 using the counting all without models strategy, and 12 out of 22 using the counting up from first strategy solved the problem correctly. None of the subjects using unidentifiable strategies either before or after instruction solved the problem correctly.

To summarize, in the first interview, before subjects had received instruction, two strategies were primarily used to solve the compare addition problem. These strategies were the counting all with models strategy which was chosen by most subjects, and the counting up from first strategy. By the second interview, a third strategy had been added. This strategy was the counting all without models strategy used primarily by subjects in the delayed and immediate groups. Before instruction, the counting all with models strategy was used by most subjects in all three treatment

groups. During the second interview, however, the counting up from first strategy was used by more subjects in the delayed and immediate groups. Subjects in the traditional group continued to rely upon the counting all with models strategy. Again, subjects solved more problems correctly after instruction than before.

Solving Compare Subtraction Problems

Compare subtraction problems involve comparing two quantities such as in the problem, "Tony has 8 marbles. Bob has 3 marbles. How many more marbles does Tony have than Bob?". This problem seemed to be the most difficult of all the problem types to solve. Table 20 shows the problem solving strategies employed before and after instruction when solving this problem.

Thirty-five subjects out of 66 did not use an identifiable strategy to solve the problem during the first interview before receiving instruction. They resorted to guessing or giving up in most cases. Many of the subjects simply gave 8 as an answer because the interviewer told them that Tony had 8 marbles. When asked how they arrived at their answer they usually said, "You told me." There was little improvement by the second interview. Twenty-eight subjects continued to used unidentifiable strategies to solve the problem.

The most commonly used strategy was the matching strategy. Twenty-three subjects used this procedure when solving the compare subtraction problem during the first

Table 20. Problem Solving Strategies Used When Solving Compare Subtraction Problems Before and After Instruction

Strategy	Time[a]	Treatment Group: Traditional	Immediate	Delayed	Total	# Correct[b]
Counting All With Models	B	0	0	0	0	--
	A	0	0	0	0	--
Counting All Without Models	B	0	0	0	0	--
	A	0	0	0	0	--
Counting Up From First	B	0	0	0	0	--
	A	0	0	0	0	--
Adding On	B	0	0	0	0	--
	A	0	2	2	4	3
Separate From	B	2	4	2	8	4
	A	7	4	3	14	5
Matching	B	7	8	8	23	9
	A	5	9	6	20	10
Basic Facts	B	0	0	0	0	--
	A	0	0	0	0	--
Other	B	13	10	12	35	0
	A	10	7	11	28	0

[a] B = before instruction
A = after instruction

[b] Total Correct Before Instruction = 13
Total Correct After Instruction = 18

interview before receiving instruction. During the second interview, 20 subjects used the matching procedure.

The second most commonly used strategy was the separate from strategy. During the first interview, 8 subjects used the separate from strategy. During the second interview, 14 subjects used the separate from strategy. Seven of those subjects belonged to the traditional group. Four subjects in the immediate and 3 in the delayed group used the separating from strategy.

During the second interview, an additional strategy not used during the first interview appeared. This strategy was the adding on strategy. Two subjects in the delayed group and 2 subjects in the immediate group used this strategy to solve the combine subtraction problem. Three of those subjects answered the problem correctly.

Most subjects interviewed before receiving instruction did not know how to solve or try to solve the combine subtraction problem (13). There was some but very little improvement by the second interview (18). Most subjects used the matching procedure both before and after instruction to solve the problem. Before instruction, 9 out of 23 subjects using the matching strategy and 4 out of 8 subjects using the separate from strategy solved the problem correctly. After instruction, 3 of the 4 subjects using the adding on strategy, 10 out of 20 subjects using the matching strategy, and 4 out of 14 subjects using the separate from strategy solved the problem correctly.

The traditional group had the same number of subjects using the separating from strategy and the matching strategy during the second interview. Before instruction, most subjects in the traditional group used the matching strategy. During the second interview, a third strategy appeared to be used that had not been previously used. That strategy was the adding on strategy and was used by 4 subjects belonging to the immediate and delayed groups. No other strategies were employed when subjects were asked to solve the combine subtraction problem.

Problem Solving Strategies Before and After Instruction

Overall, it seems that subjects moved from less efficient or inappropriate strategies before instruction to more efficient and appropriate strategies after instruction. This seemed especially true for subjects in the immediate and delayed groups. For example, when solving combine addition problems, subjects in the immediate and delayed groups moved from using the counting all with models strategy to the counting all without models and counting up from first strategy, both of which are considered to be more efficient strategies (Carpenter & Moser, 1982). In the traditional group, approximately the same number of subjects using the counting all with models strategy before instruction used this same strategy after instruction. For all problems, there seemed to be fewer subjects in the traditional group using new and more efficient strategies in the second interview.

More subjects used an identifiable strategy after instruction than before instruction. Thus, less "guessing" occurred after the instructional period. There was only one incident in which there were more subjects using an unidentifiable strategy after instruction than before instruction. When solving the combine subtraction problem, there were 3 more subjects in the traditional group using an unidentifiable strategy after instruction (7) than before instruction (4). In all other cases, subjects moved from using an unidentifiable strategy such as guessing to an identifiable strategy.

Strategies such as counting all without models, counting up from first, adding on, and basic facts were used more often in the second interview than in the first. The strategies used most in the first interview were the counting all with models strategy and the separate from strategy. This could suggest that subjects were moving from less efficient strategies to more efficient strategies either learned through the instructional process (although none were intentionally taught as a means for solving certain problems) or which were developed independently or from observing other models, i.e. teachers and peers.

CHAPTER 5
SUMMARY, CONCLUSIONS, DISCUSSION, LIMITATIONS, AND IMPLICATIONS

The effects of three instructional approaches designed to introduce first graders to addition and subtraction symbols were examined as a part of this study. The time at which students were introduced to addition and subtraction symbols varied in each instructional group. One group was introduced to symbols before story problems (traditional group), another group immediately following each story problem (immediate group), and the third group after several weeks of solving story problems (delayed group). The problem solving strategies used by first graders to solve addition and subtraction story problems both before and after instruction were examined as a part of this study as well.

A summary of the results of this study is presented in this chapter followed by conclusions, discussion, and observations. Finally, limitations and implications of the study are discussed.

Summary of the Study

It was hypothesized that the time of introduction to addition and subtraction symbols would have no effect on

first graders' ability to solve addition and subtraction number sentences, produce and interpret addition and subtraction number sentences, and solve addition and subtraction story problems. A pretest - posttest comparison group design was used to test this hypothesis using a sample of 66 first grade students. The study lasted for 7 weeks.

Pretests administered during the first week of the study measured subjects' ability to solve, produce, and interpret addition and subtraction number sentences as well as solve addition and subtraction story problems. In addition, each subject was clinically interviewed to determine what strategies were used in solving addition and subtraction story problems. Posttests identical to the pretests were administered the week following the 5-week treatment period. Four weeks following the completion of the study, subjects were given a second set of posttests measuring their ability to solve addition and subtraction number sentences, produce and interpret addition and subtraction number sentences, and solve addition and subtraction story problems to determine if differences which existed immediately after the treatment period continued to exist 4 weeks later. These tests were identical to both the pretests and posttests. No clinical interviews were administered, however, 4 weeks following the completion of the treatment period.

Data collected from the pretests were analyzed using an analysis of variance procedure to see if there were initial

differences among treatment groups. Both sets of posttests were statistically analyzed using an analysis of covariance procedure with pretest scores as the covariate. To find specific differences among means, the Bonferroni technique of multiple comparisons was performed.

Conclusions and Discussion

The results of this study would lead to the conclusion that the time of introduction of symbolization does have an effect on first graders' meaningful learning of written addition and subtraction symbols. Following is an elaboration on this conclusion and the findings of this study. The discussion is divided into three parts. In the first two parts, the first graders' understanding of addition and subtraction symbols and ability to solve story problems are discussed. In the third part, the results of the clinical interviews are discussed.

Understanding Addition and Subtraction Symbols

On all measures of symbol understanding, (solving number sentences, producing number sentences, and interpreting number sentences) there were significant differences among treatment groups. When testing Hypothesis 1 which stated that the time of introduction to addition and subtraction symbols would have no effect on first graders' ability to solve addition and subtraction number sentences,

the immediate group was found to have performed significantly better than the traditional and delayed groups on the posttest given immediately after the treatment period, although on the posttest given 4 weeks later, no significant differences were found to exist among treatment groups.

Results of the measure of ability to produce number sentences were used to test Hypothesis 2 stating the the time of introduction to addition and subtraction symbols would have no effect on first graders' ability to produce addition and subtraction number sentences. Statistical analyses revealed that the immediate group performed significantly better than the traditional and delayed groups on the posttest given immediately after the treatment period. On the posttest given 4 weeks later, however, both the immediate and delayed groups performed significantly better than the traditional group.

When testing Hypothesis 3 which stated that the time of introduction to addition and subtraction symbols would have no effect on first graders' ability to interpret addition and subtraction number sentences, the delayed group performed significantly better than the traditional group on the posttest measure of ability to interpret number sentences given just after the treatment period. On the posttest given 4 weeks later, both the delayed and immediate groups performed significantly better than the traditional group.

Finally, on the measure of total symbol understanding, both the immediate and delayed groups performed significantly better than the traditional group on the posttest given immediately following the treatment period as well as on the posttest given 4 weeks later. Thus the null hypothesis stating that the time of introduction to addition and subtraction symbols would have no effect on first graders' ability to understand addition and subtraction symbols (Hypothesis 4) was rejected.

These findings seem to indicate that introducing symbols after experiences in solving story problems verbally can assist in helping first graders to better understand addition and subtraction symbols. Whether symbols are introduced immediately after or several weeks after the introduction of story problems does not seem to be crucial in helping children understand symbols. What does seem important, however, is that students are provided with experiences in solving problems orally or with the assistance of manipulatives and role play before introducing the numerical and operational symbols and that symbols then be introduced as a means of representing the already solved story problems.

Hiebert (1984) states that children understand a symbol when they know what that symbol represents. When addition and subtraction symbols are presented first, it may be difficult to view them as "standing for" something since what they represent has not yet been introduced. For

example, it would be difficult to understand an abbreviation for a word if the whole word had not yet been introduced. Thus, symbols may come to be viewed as separate entities in and of themselves with little connection to anything else. This could possibly have been the case with subjects in the traditional group.

During the course of instruction, the traditional group seemed more concerned with manipulating the symbols and getting a "right answer" than with establishing a link between story problems and the number sentences representing them. When number sentences were presented to the traditional group, subjects immediately began solving the number sentences by counting on their fingers or moving objects to arrive at an answer. The story told to go with the number sentence seemed of little interest to them. The symbols seemed to distract them so that they were unable to concentrate on the accompanying story problem being told. As a result, there was little opportunity to make the connection between the symbol (number sentence) and what was representing the symbol (story problem).

Subjects in the immediate and delayed groups had this opportunity, however, due to the fact that the story problem was always told before introducing symbols. In order to solve the problem and eventually write the number sentence, subjects had to listen carefully to the story and understand which symbol represented which part of the story. Thus, they were constantly involved in making the link between the

symbol and the process. Subjects in the traditional group had no real reason to concentrate on the story problems being told because they already had the essential elements of the problem presented. It is similar to reading a mystery. If the mystery is solved on the first page of the book, there is very little reason to read the rest of the book because much of the process involved in reading a mystery is gathering evidence to arrive at a solution.

The immediate and delayed groups differed on the posttests given immediately after the treatment period measuring ability to produce and interpret number sentences. On the posttests given 4 weeks after the treatment period, their performance on those measures was statistically equal while statistically better than the traditional group. The fact that subjects in the immediate group were better at producing number sentences at the time of the posttest may have been due to the fact that they had had more practice in the actual writing of number sentences. The delayed group had only been introduced to number sentences a week earlier. By the time students were given the second set of posttests, subjects in the delayed group had had four weeks to practice writing and solving number sentences. This could account for their improved performance on the measure of ability to produce number sentences.

On the posttest given immediately following the treatment period, the delayed group had a significantly higher score than the immediate and traditional groups on the measure to interpret number sentences ($p = .041$). On

the posttest given 4 weeks later, scores for the delayed and immediate groups did not significantly differ from each other but were significantly higher than the mean score for the traditional group.

Developing meaning for symbols is an instructional task that is important at all levels of mathematics yet is often neglected (Hiebert, 1982). Much time is spent teaching children how to manipulate symbols but little time is devoted to teaching what those symbols actually mean. The best way to introduce children to symbols and help them to understand their meanings is not known. The findings of this study, however, provide additional information pertaining to this instructional process. The argument that verbal problems provide an excellent context in which to introduce symbols (Campbell, 1978; Hiebert, 1984) is supported by the findings of this study. In addition, the findings of this study support the recommendation that symbols be introduced following experiences with story problems and as representations of those story problems (Campbell, 1978). This study does not provide conclusive evidence suggesting whether it is better to introduce symbols immediately following or several weeks after solving story problems.

Solving Story Problems

On both the posttest given immediately after the treatment period and the posttest given 4 weeks later, the delayed and immediate groups performed significantly better

than the traditional group on the measure of ability to solve story problems thus rejecting Hypothesis 5 stating that the time of introduction to addition and subtraction symbols would have no effect on first graders ability to solve story problems. This could again be due to subjects in the immediate and delayed groups being more interested in story problems than the traditional groups because they were not distracted by being first introduced to symbols.

Not only were subjects in the immediate and delayed groups better at solving story problems, but they also produced more elaborate and detailed story problems than those in the traditional group. As part of the instructional process, each treatment group had the opportunity to make up its own story problems. The traditional group created group-composed stories to go with number sentences presented to them. The immediate and delayed groups made up story problems without the introduction of number sentences first. For example, they might be asked to make up a story in which a girl has 4 dolls and ends up with more than 4 dolls. This same direction would be given to subjects in the traditional group but the number sentence would already be visible.

For this particular task, the story problem emerging from the traditional group was simply, "A girl had 4 dolls and got some more and ended up with 6 dolls." The story from the immediate group involved each doll acquiring a sister so that the girl ended up with 4 more dolls making

8 in all. Subjects in the delayed group described the girl in the story opening birthday presents until she opened her last two presents, each of which contained a doll, so that she ended up with 6 dolls instead of 4. This is just one of several examples showing the difference in the quality of story problems which typically emerged from the traditional group versus those of the immediate and delayed groups.

It is also interesting to note that across all treatment groups, scores were lower on the measure of ability to solve story problems given 4 weeks following the completion of the treatment period than on the posttest given immediately after the treatment period. This could possibly have been due to the fact that since the completion of the interventions, the regular classroom teachers had not provided any further experiences in solving story problems. The majority of the mathematics instruction they received after the interventions and before the second set of posttests were administered dealt with solving and writing answers to addition and subtraction number sentences. The subsequent lower scores could be an indication of the importance of providing children with continued practice in solving story problems.

Clinical Interviews

Contrary to the findings of Carpenter et al. (1983), more problem solving strategies were used during the second interview, after the instructional period, than in the first interview. Carpenter et al. (1983) examined the problem

solving strategies used by first graders before and after receiving instruction and found that first graders had a tendency to rely upon single problem solving strategies after receiving instruction rather than using a variety of problem solving strategies as was done before receiving instruction. In the present study, however, there were more problem solving strategies used after instruction than before instruction. This is partially a result of more students using identifiable strategies by the second interview instead of simply guessing. Many students who had not had any experience in listening to or solving story problems guessed answers during the first interview rather than trying to solve the problems.

Another possible reason that additional strategies were used after instruction than before, is that new problem solving strategies were developed as a result of the instructional process. One strategy in particular was the basic facts strategy. Because students had the opportunity to become familiar with and practice using basic addition and subtraction facts as a result of the instruction they received, they were then able to employ the basic facts strategy during the posttests which before had been impossible to employ.

The increase in the number of strategies used after instruction may also be due to the fact that many more subjects began using more efficient problem solving strategies perhaps as a result of their repeated experiences

with and practice in solving addition and subtraction problems. This was especially true for subjects in the immediate and delayed groups. Subjects in the traditional group had a tendency to continue using the strategies they had used before receiving instruction. For example, when solving combine addition problems, more subjects in the immediate and delayed groups began using the counting from first strategy rather than the counting all with models strategy. Most subjects in the traditional group, however, continued using the counting all with models strategy. There were only three cases out of the entire sample in which students moved from using more to less efficient problem solving strategies.

The difference in the findings of this study and those of Carpenter et al. (1983) may also be due to the differences in the design and implementation of the two studies. In this study, subjects were interviewed before and after a 5-week instructional period. In the study by Carpenter et al., however, subjects were interviewed before and after an entire year of mathematical instruction. Students were then interviewed again after 2 years of instruction and finally after 3 years of instruction. If subjects in the present study had been interviewed after a longer instructional period, the findings may have been very different.

In addition, the type of instruction received by the subjects in the two studies varied which very likely may

have affected their choice of problem solving strategies. All subjects in the study by Carpenter et al. received the same kind of instruction which more than likely amounted to a few minutes of direct instruction each day followed by having students complete worksheets or workbook pages related to the instructional topic. Subjects in the present study, received one of three different instructional approaches each of which never directly taught specific problem solving strategies and constituted a 25-minute block of time each day. Thus, the amount of time each day that subjects actually received instruction from a teacher was probably greater for subjects of the present study than for those in the study conducted by Carpenter et al. Because the type of instruction administered in the present study was controlled, the quality of instruction may have been better as well.

Combine addition problems. When solving combine addition problems, the majority of students employed a correct strategy or strategy which when used properly, could lead to a correct answer. Most subjects used the counting all with models strategy both before and after instruction across all treatment groups. After instruction, there was an increase in the number of subjects in the immediate and delayed groups using the counting all without models strategy, the counting up from first strategy, and the basic facts strategy. Most subjects in the traditional group continued using the counting all with models strategy.

Subjects in the traditional group tended to rely upon this strategy for solving number sentences as well.

Combine subtraction problems. Before and after instruction, more subjects across all treatment groups used the separate from strategy to solve the combine subtraction problem than any other strategy. Two subjects used an incorrect strategy (counting all with models) before instruction probably because they understood the problem to be an addition problem. There were, again, more strategies used after instruction than before instruction unlike the findings of Carpenter et al. (1982). However, in the study by Carpenter et al. (1982), students were taught to solve story problems by first identifying whether a story problem was an addition or subtraction problem before solving the problem. This process seems to encourage children to regard all subtraction problems as similar and to solve them using a single strategy such as the separate from strategy (Hiebert, 1984). Subjects in the present study were never directly instructed as to how to solve story problems.

Separate subtraction problems. More subjects solved the separate subtraction problem correctly than any other subtraction problem. The strategy used by the majority of students was the separate from strategy. This strategy seems to be a logical strategy to use since it is implied in the context of the story problem. The only other strategies used were the adding on and basic facts strategy. Both were used by more students after instruction than before

instruction. It is somewhat surprising that so many subjects used the adding on strategy because it is not implied in the structure of the problem and is a strategy which is usually directly taught to students as a part of faster way to solve subtraction problems. The majority of subjects using the adding on strategy solved the problem correctly.

Compare addition problems. Before receiving instruction, more subjects across all treatment groups used the counting all with models strategy than any other strategy. After receiving instruction, however, more subjects in the immediate and delayed groups used the counting up from first strategy. Subjects in the traditional group continued to rely upon the counting all with models strategy. The same strategies were used by subjects when solving combine addition problems. It seems that children view these two problems as being similar. It also seems that when solving addition problems, subjects tend to rely most upon counting strategies. These findings are similar to the findings of Carpenter et al. (1982).

Compare subtraction problems. The compare subtraction problem was the most difficult of all problems to solve correctly. Many subjects both before and after the interviews used unidentifiable problem solving strategies and solved the problem incorrectly. Only two strategies were used before instruction (matching and separate from). The adding on strategy was used by subjects in the immediate

and delayed groups after instruction. Other students simply named one of the addends of the problem as the answer. For example, when told that "Bill had 9 marbles and Kathy had 3", many subjects gave 9 as the answer when asked how many more marbles Bill had than Kathy. The actual wording of the problem may have been confusing to children. If asked how many more marbles Kathy would need to have the same number of marbles as Bill, more children may have been able to solve the problem correctly or at least employ an appropriate problem solving strategy.

It is interesting to note that both before and after instruction, there were no subjects who solved problems by first writing down a number sentence to correspond with the story problem then solving the number sentence. It is possible that this method for solving story problems is unnatural to young children and may not be helpful to students when solving story problems. Many primary teachers and textbooks teach students to solve problems in this manner. When students begin solving problems with large quantities, writing number sentences may be useful in remembering and adding or subtracting those quantities. This would be especially true if there were more than two quantities represented in the problem. Teaching students to use number sentences to solve story problems at the introductory level, however, may not be seen as useful or logical to young children.

Observations

Several observations were made while implementing this study which should be mentioned. First it was observed that as the study progressed, fewer students relied upon manipulatives to help them in solving problems. During the pretest, 17 subjects in the traditional group, 18 subjects in the immediate group, and 19 subjects in the delayed group used manipulatives to help them in solving addition and subtraction number sentences and story problems. When administering the first set of posttests, 13 in the traditional group, 10 in the immediate group, and 9 in the delayed group chose to use manipulatives to help them solve number sentences and story problems.

Second, during the posttest given immediately after the treatment period measuring the ability to solve number sentences, several subjects in the immediate and delayed groups were heard whispering story problems to themselves to go with the number sentences. For example, one subject was heard to say, "3 apples, got 2 more, 5 in all," when solving the number sentences, 3 + 2 = __. No subjects in the traditional group were heard whispering story problems to go with number sentences they were solving.

Finally, it was observed that much more discussion took place in both the immediate and delayed groups when solving problems. This seemed especially true in the delayed group.

Although students in all treatment groups were allowed and encouraged to discuss how problems were solved, students in the immediate and delayed groups seemed to have more to discuss than in the traditional group. Once again, students in the traditional group seemed too busy actually solving the number sentences (counting fingers and objects) to participate in discussions.

Limitations of the Study

There are several circumstances of this study which limit the generalizability of the findings. First, a small sample of subjects was used coming from one school in one city and state. It would be presumptuous to generalize the findings of this study to all first graders across the country. Similar studies would first need to be conducted in varying sections of the country possibly using a larger number of subjects although this study did include subjects of a variety of races, abilities, and socioeconomic levels.

Another possible limitation of this study was the use of intact classrooms for treatment groups. Subjects were first pretested to see if initial differences existed and pretest scores were used as a covariate when analyzing data to statistically control for initial differences. Nevertheless, there may possibly have been differences in the groups not detected by the pretest measure which may

have affected the results of the study, for example, the attitude of the regular classroom teacher, the general attitude or behavior of the group, etc.

The length of the study may also have been a limitation of the study. The treatment period only lasted 5 weeks for 25 minutes a day. Results of the study might have differed, for example, if the treatment period had been extended so that the delayed group would have had more than just 1 week to use addition and subtraction symbols.

This study examined the effect of only three instructional methods on the learning of addition and subtraction symbols using small numbers. This prevents the generalizability of the conclusions of the study to other topics in mathematics and to other grade levels. Further research using a variety of grade levels and mathematical topics would allow conclusions to be made concerning other topics and grade levels.

A further limitation of the study was the inability to control for other possible confounding variables which might have affected the results of this study. One possible variable was discipline. The immediate class, for example, was generally more attentive and better behaved than the delayed and traditional group. Other possible confounding variables include room arrangement and organization, learning styles of individual students, student motivation, and motivation and attitude of the classroom teacher.

Implications of the Study

Taking into account the limitations just discussed and recalling the fact that all subjects had prior mastery of basic numeration concepts, the following curricular implications can be made as a result of this study.

1. There appears to be some benefit in providing first graders with experiences in solving addition and subtraction story problems before introducing addition and subtraction symbols (number sentences). This seems to help provide for a better understanding of the symbols themselves as well as a better ability to solve story problems and employ appropriate problem solving strategies.
2. Students should be provided with opportunities to analyze story problems and develop their own means of solving problems before being introduced to symbolic representations of problems and a prescribed strategy for solving those problems.
3. Students should be given opportunities to not only solve story problems but translate story problems into number sentences as a representation of those story problems.
4. Students should be provided with continual experiences in solving story problems in addition to using story problems to introduce addition and subtraction symbols.

Suggestions for Further Research

There still remain some unanswered questions concerning the way children are introduced to mathematical symbols. While it was found, as a result of this study, that introducing symbols after experiences in solving verbal story problems helped children understand addition and subtraction symbols, it is not known whether this would be true when introducing other operational symbols. It is also unknown whether the results of this study would be the same for a different population of first graders. By repeating the study with a different and larger population of first graders, generalizations could be made to all first graders with more confidence.

Addition and subtraction were introduced simultaneously in this study, yet students still had more difficulty in solving subtraction problems than addition. A common error in solving addition and subtraction number sentences was assuming all problems were addition problems even though both addition and subtraction had been introduced together. The reasons for these phenomena should be further explored as well as what in the instructional process may be causing these phenomena.

Results from the investigation of problem solving strategies used before and after instruction differed from the results of investigations conducted by Carpenter, Hiebert, and Moser (1982). Reasons for these differences

should be further examined. For example, there is a need for a study which examines the differences in the problem solving strategies chosen and used by pupils who have been directly taught how to use strategies for specific problems versus pupils allowed to develop their own problem solving techniques.

There is also a need to further examine how problem solving processes and strategies are developed. In this study, most subjects moved from using less efficient to more efficient problem solving strategies. Only three subjects moved from using more to less efficient problem solving strategies. Whether or not students progress through similar problem solving strategies and in the same order needs to be investigated. For example, do most subjects move from counting all with models, to counting all without models, to counting from first, counting from larger, and finally to the use of basic facts? The possibility of a developmental hierarchy may be indicated by the results of this study and needs to be researched.

Concluding Remarks

The primary purpose of mathematics instruction is to help children see meaning and order in everyday situations (Payne, 1975) while providing them with tools to cope with "real-world" situations. Unfortunately, however, schools often fail to demonstrate the role of mathematics in the

outside world and students then experience difficulties when faced with "real-world" problems to solve (Kamii, 1982). It is crucial that schools begin to relate the meaning of real world experiences with arithmetic concepts and symbols which represent those experiences. It is necessary, therefore, to develop appropriate instructional strategies designed to introduce young children to arithmetic and the associated symbols.

Three instructional approaches were examined in this study and were all found to be effective in teaching young children to add and subtract. Being able to add and subtract, however, does not mean that children will be able to solve "real-world" problems which require addition or subtraction. This was made evident by the poor performance of the traditional group on the measure of ability to solve story problems. It also does not guarantee that students will be able to understand symbolic expressions or see their relationship to the mathematical concepts underlying them. This, again, was made obvious by the inability of subjects in the traditional group to correctly interpret and produce number sentences.

Verbal problems appear to be a good context in which to introduce the concepts and symbols of addition and subtraction. Simply providing students with experiences in solving verbal or story problems is not sufficient, however. Subjects in the traditional group were provided with experiences in solving story problems as were subjects in

the immediate and delayed groups. However, subjects in the traditional group did not perform as well as subjects in either the immediate or delayed groups on measures of symbols understanding and problem solving. How and when story problems are used in the instructional process seems to be very important.

In the traditional group, number sentences were introduced first with an accompanying story problem. By introducing the number sentence first, the emphasis of the instruction was directed at solving the number sentence rather than understanding and solving story problems. Students could have naturally assumed that the number sentences were more important than the associated story problems and what was most important was producing the correct answer to those number sentences. Story problems were "added extras" to make solving number sentences more fun and possibly easier.

In the immediate and delayed groups, on the other hand, story problems were introduced first with accompanying number sentences introduced later. Thus the emphasis of instruction was to solve story problems first followed by a representation of story problems in the form of number sentences. As a result, students could have viewed solving story problems as more important than producing answers to number sentences which is more in line with the primary goal of mathematics instruction.

The emphasis of mathematics instruction should be on the development of sound mathematical concepts and an understanding of how those concepts relate to the outside or "real" world. Story problems are descriptions of events that could occur in the real world. By emphasizing the solving of story problems rather than the solving of number sentences, children can begin to form that important link between mathematics and the outside world.

APPENDICES

APPENDIX A
SUMMARY TABLES OF STATISTICAL ANALYSES

Table A.1. ANOVA Summary Table for Pretest Measure of Ability to Solve Addition and Subtraction Number Sentences

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	3.91	1.95	0.24	0.78
Error	63	519.68	8.24		
Total	65	523.59			

Table A.2. ANOVA Summary Table for Pretest Measure of Ability to Produce Addition and Subtraction Number Sentences

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	3.12	1.56	0.10	0.90
Error	63	985.31	15.63		
Total	65	988.43			

Table A.3 ANOVA Summary Table for Pretest Measure of Ability to Interpret Addition and Subtraction Number Sentences

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	8.45	4.22	0.77	0.46
Error	63	346.81	5.50		
Total	65	355.26			

Table A.4 ANOVA Summary Table for Pretest Measure of Total Symbol Understanding

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	31.12	15.56	0.20	0.81
Error	63	4873.54	77.35		
Total	65	4904.66			

Table A.5 ANOVA Summary Table for Pretest Measure of Ability to Solve Addition and Subtraction Story Problems

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	9.13	4.56	0.19	0.82
Error	63	1507.90	23.93		
Total	65	1517.03			

Table A.6 ANCOVA Summary Table for Posttest Measure of Ability to Solve Addition and Subtraction Number Sentences Given Immediately Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	84.22	42.10	4.92	0.01
Error	63	539.31	8.56		
Total	65	623.53			

Table A.7. ANCOVA Summary Table for Posttest Measure of Ability Produce Addition and Subtraction Number Sentences Given Immediately Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	167.18	83.59	7.56	0.001
Error	63	696.59	11.05		
Total	65	863.77			

Table A.8. ANCOVA Summary Table for Posttest Measure of Ability to Interpret Addition and Subtraction Number Sentences Given Immediately Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	122.30	61.15	3.37	0.04
Error	63	1144.31	18.16		
Total	65	1266.62			

Table A.9. ANCOVA Summary Table for Posttest Measure of Total Symbol Understanding Given Immediately Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	1121.18	560.59	3.85	0.02
Error	63	9162.40	145.43		
Total	65	10283.59			

Table A.10. ANCOVA Summary Table for Posttest Measure of Ability to Solve Addition and Subtraction Story Problems Given Immediately Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	384.37	192.18	29.47	0.0001
Error	63	410.90	6.52		
Total	65	795.27			

Table A.11. ANCOVA Summary Table for Posttest Measure of Ability to Solve Addition and Subtraction Number Sentences Given Four Weeks Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	40.93	20.46	1.50	0.23
Error	63	860.04	13.65		
Total	65	900.97			

Table A.12. ANCOVA Summary Table for Posttest Measure of Ability Produce Addition and Subtraction Number Sentences Given Four Weeks Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	209.30	104.65	7.05	0.001
Error	63	934.81	14.83		
Total	65	1144.12			

Table A.13. ANCOVA Summary Table for Posttest Measure of Ability to Interpret Addition and Subtraction Number Sentences Given Four Weeks Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	180.03	90.01	6.43	0.002
Error	63	882.59	14.00		
Total	65	1062.62			

Table A.14. ANCOVA Summary Table for Posttest Measure of Total Symbol Understanding Given Four Weeks Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	842.81	421.40	4.94	0.01
Error	63	5375.18	85.32		
Total	65	6218.00			

Table A.15. ANCOVA Summary Table for Posttest Measure of Ability to Solve Addition and Subtraction Story Problems Given Four Weeks Following the Treatment Period

SOURCE	DF	SS	MS	F	PR > F
Treatment	2	300.21	150.10	8.77	0.0004
Error	63	1078.04	17.11		
Total	65	1378.25			

APPENDIX B

SAMPLE RESEARCH INSTRUMENTS

MEASURE OF ABILITY TO SOLVE STORY PROBLEMS

Pretest / Posttest (names will change for posttest)

1. (Combine - Subtraction)

   Kristin had 6 blocks.

   Some were red and some were blue.

   Two blocks were red.

   How many blocks were blue?

2. (Separate - Subtraction)

   Tim had 5 pieces of candy.

   He gave 2 pieces to Martha.

   How many pieces of candy does Tim have left?

3. (Combine - Addition)

   There were some kids on the playground.

   Three were boys and four were girls?

   How many kids were there altogether on the playground?

4. (Compare - Addition)

   Katie has four toy cars.

   Robin has five more cars than Katie.

   How many cars does Robin have?

5. (Compare - Subtraction)

Bill has 9 marbles.

Kathy has 3 marbles.

How many more marbles does Bill have than Kathy?

PROBES - IF NEEDED

1) Show samples of + and - number sentences	"These are some number sentences." "Can you write a number sentence to go with what I did with the blocks?"
2) Repeat the action again with the blocks.	Repeat what was said before when manipulating the blocks
	"Try to write a number sentence you think would go with what I just did."

(Examiner's blocks should be one color and student's blocks another color to help child remember how many each person had initially.)

Task 3: Interpreting Number Sentences

Show the following number sentence:

6 + 3 = 9

Say, "Can you use the blocks to show me how to do this problem?

If child doesn't want to or responds "no", say,

"Can you draw a picture to go with this problem?"

If child doesn't want to or responds "no", say,

"Can you tell me a story to go with this problem?"

Make a record of all responses and actions.

Repeat the above procedures for the following number sentence:

5 - 3 = 2

APPENDIX C

FORMAT FOR CLINICAL INTERVIEWS

1. Read the following problem 2 times.

   John had 4 cookies.

   His mom gave him 3 more cookies.

   How many cookies does John have altogether?

2. Wait for a response.

3. If no response, say:

   "Well, let's think. John had FOUR cookies. Then his mom gave him THREE MORE. Now how many does he have altogether?"

4. Response from child.

5. "O.K. How did you figure that out?" (Use a very nonthreatening tone of voice. Act as if you're really curious and think it's "neat".

6. Wait for response. If says "I don't know", try to get to explain, show with manipulatives, or show with a picture.

7. Record response on audiotape and write notes as to physical behavior (counting on fingers, etc.)

8. Repeat the above procedures using different story problems.

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BIOGRAPHICAL SKETCH

Suzanne McWhorter Colvin, daughter of William Horace and Kathleen Story McWhorter, was born August 22, 1960, in Valencia, Venezuela. At six years of age, she moved with her family to the United States. She earned a high school diploma from Andalusia High School, Andalusia, Alabama, in 1978. In 1979, she received an Associate of Arts degree (with honor) from Lurleen B. Wallace State Junior College in Andalusia, Alabama. In December, 1981, she received a Bachelor of Science degree in education (with highest honor) from Auburn University, Auburn, Alabama. She entered the Graduate School of Auburn University in the spring of 1982. In December, 1983, she received a Master of Science degree in education. She transferred to the University of Florida to pursue a Doctor of Philosophy degree in education. In May, 1987, she was awarded the degree of Doctor of Philosophy in education. She was married to Daniel Lamar Colvin, December 19, 1981.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Suzanne L. Krogh

S. L. Krogh, Chairperson
Associate Professor of Instruction & Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Alan Agresti

A. G. Agresti
Professor of Statistics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

E. J. Bolduc

E. J. Bolduc
Professor of Instruction & Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

D. A. Knauft

D. A. Knauft
Associate Professor of Agronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Dorene D Ross

D. D. Ross
Associate Professor of Instruction & Curriculum

This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Dean, College of Education

Dean, Graduate School
May, 1987